AUCTIONS VS. NEGOTIATIONS: THE CASE OF FAVORITISM

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Abstract. We compare two commonly used mechanisms in public procurement: auctions and negotiations. The execution of the procurement mechanism is delegated to an agent of the buyer. The agent has private information about the buyer’s preferences and may collude with one of the sellers. We provide a precise definition of both mechanisms and show – contrary to conventional wisdom – that an intransparent negotiation yields a higher buyer surplus than a transparent auction for a range of parameters. In particular, there exists a lower bound on the number of sellers such that the negotiation yields a higher buyer surplus and is more efficient with a probability arbitrary close to 1 in the parameter space.

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1. Introduction

Auctions are believed to be transparent mechanisms and hence less prone to favoritism than private negotiations. For instance, Paul Klemperer (2000) argues that ",..., allocation by bureaucrats leads to the perception - if not the reality - of favoritism and corruption. In fact some governments have probably chosen beauty contests [over auctions] precisely because they create conditions for favoring “national champions” over foreign competitors. This is unlikely to benefit consumers and taxpayers.”

The perception that auctions are transparent mechanisms stems from the fact that auctions are executed publicly, whereas negotiations are conducted privately. Hence, in an auction all relevant parameters and rules have to be defined before the bidders submit their offers and it is apparent whether the implemented procedures have been followed. In a

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1More recently Subramanian (2010) argues: “Auctions are more transparent processes than private negotiations, so if transparency is important, an auction is better. This is the reason that most public procurement contracts [...] are done through auctions, particularly when the government is looking to defuse criticisms of corruption or favoritism.” Moreover, Martin Wolf (2000) argues that “it [the auction] is the fairest [mechanism] because it ensures that the economic value goes to the community, while eliminating the favoritism and corruption inherent in bureaucratic discretion.”
negotiation – on the other hand – it is impossible to reconstruct the decision process and only the final decision becomes public.

However, public scrutiny does not imply that auctions are favoritism proof, as the parameters and procedures of an auction may be chosen in a way that benefits one of the sellers before the auction has even started. Moreover, even though a negotiation is conducted privately, the final outcome of the process has to be rationalized to the public after all offers have been collected. Thus, some public scrutiny cannot be avoided in a negotiation.²

This paper focuses on the definition and comparison of auctions and negotiations in the presence of favoritism. For both processes we consider a procurement setting with sellers that are horizontally differentiated with respect to the specification of the procured project.³ Buyer surplus depends not only on the final price but also on the implemented specification. The buyer has to delegate the execution of either process to an agent who privately observes the specification preference of the buyer and colludes with one – exogenously chosen – seller.⁴ The agent maximizes the surplus of his preferred seller and has a weak preference for honesty, i.e. prefers not to manipulate either process if his preferred seller cannot strictly benefit from manipulation.

We start our analysis by arguing that the main difference between auctions and negotiations in terms of transparency is that in an auction public scrutiny is imposed before the agent collects the offers of the sellers, whereas in the negotiation public scrutiny is imposed after collecting the offers. Hence, public scrutiny in an auction restricts the choice of the process, whereas in the negotiation public scrutiny merely places restrictions on the final decision of the agent. In our set-up, the manipulation power of the agent stems from the fact that the preferred specification of the buyer is private knowledge to the agent. Thus, public scrutiny in the auction implies that the implemented procedure has to be optimal given some feasible specification.⁵ In the negotiation, public scrutiny implies that in the

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²This argument generalizes to private auctions and negotiation. Even though, private procurement is not conducted publicly the managers still have to answer to the shareholders of the procuring company.
³For example, in engineering plastics (Polyamide, Polycarbonate) there is a trade-off between rigidity and flexibility. Different grades of plastics from different suppliers have different characteristics. Prior to the procurement auction the project team decides on the for the project optimal specification (i.e. relation of rigidity and flexibility).
⁴The assumption that the agent colludes with one specific seller resembles many real-life situations in public procurement. For example, Laffont and Tirole (1991) argue: “There has been much concern that the auction designer may prefer or collude with a specific buyer. And indeed most military or governmental markets acquisition regulations go to a great length to impose rules aimed at curbing favoritism. Similarly, the European Economic Comission, alarmed by the abnormally large percentage (above 95% in most countries) of government contracts awarded to domestic firms is trying to design rules that would foster fairer competition between domestic and foreign suppliers and would fit better than recent experience with the aim of fully opening borders ...”
⁵In this case the agent can claim that this specification is the true specification of the buyer and that the procedure is optimal.
end the winning seller must have offered the lowest price at some feasible specification.\footnote{In this case the agent can claim that this is the true specification of the buyer.}

How this price was achieved is not salient to the public.

We proceed by precisely defining the resulting mechanisms and comparing them in terms of buyer surplus and efficiency. We find that if both the auction and the negotiation are manipulated, the buyer is better off with the auction, as the optimal auction discriminates against the (manipulated) specification. However, this does not imply that the auction performs better in general. One of our main insights is that the decision whether to manipulate the auction is different from the decision whether to manipulate the negotiation. In the auction, the decision to manipulate has to be taken before the bidders submit their offers, whereas in the negotiation, the decision to manipulate can be taken after the bidders have submitted their offers. Hence, the agent (almost) always manipulates the auction, whereas in the negotiation, the decision to manipulate depends on the realized costs and specifications of the sellers.

To get some intuition for this result, recall that in the negotiation the agent can observe the offers of the sellers before public scrutiny forces him to reveal the specification on which his allocation decision is based. Thus, the preferred specification of the buyer is only distorted if the favorite seller can benefit from the distortion ex-post. It follows that if the favorite seller turns out to be relatively weak, the specification is set optimally and the project is allocated efficiently among the honest sellers. In the auction, the details of the process have to be set prior to collecting the offers. Therefore, the auction is manipulated whenever the favorite seller can profit from manipulation ex-ante. Thus, the preferred specification is distorted even if the favorite seller is relatively weak.

We show that with two sellers the auction always generates a higher buyer surplus. If the number of bidders is small but above two, either of the processes may generate the higher buyer surplus depending on the initial specifications of the sellers. However, if the number of sellers increases, the negotiation outperforms the auction with probability close to one in the specification space.

Beyond the ranking of buyer surplus, we find that with an increasing number of sellers the negotiation is also more efficient than the auction. However, if the number of sellers is low, the ranking of the mechanism with regard to efficiency is ambiguous and depends on the chosen parameters. Interestingly, the favorite seller always prefers the negotiation over the auction mechanism. Thus, only the regular sellers may profit if an auction is used.
Relation to the literature. One of the main contributions of this paper is that it brings together two strands of literature: the literature on favoritism in auctions, and the literature on the comparison of auctions and negotiations.

In most cases favoritism enters auctions through two different channels. First, the auctioneer can favor a seller by allowing him to adjust his bid in a first-price auction after observing all of the competing bids (“right of first refusal” or bid rigging). In this case the final allocation will be inefficient and the surplus of the buyer diminishes (Burguet and Perry, 2007; Menezes and Monteiro, 2006; Lengwiler and Wolfstetter, 2010). In our model, the auction is undertaken under public scrutiny. Thus, such a form of bid rigging can not occur in the auction. In the negotiation bid rigging is possible as only the final outcome is subject to public scrutiny. Second, the auctioneer can manipulate the quality assessment of his favorite seller. This case is analyzed in Laffont and Tirole (1991), Burguet and Che (2004), and Celentani and Ganzuza (2002). We take slightly a different approach in assuming that the agent may misrepresent the preferences of the buyer rather than the quality assessment of the seller. This implies that favoritism not only distorts the mechanism for the favorite bidder but may also distort the allocation among the honest bidders.

The second strand of literature is concerned with the comparison of auctions and negotiations. Bulow and Klemperer (1996) show that a simple auction with one additional bidder leads to higher revenues than the best mechanism without this bidder. The result by Bulow and Klemperer (1996) is often used to argue in favor of auctions. However, in case the number of bidders is not an issue, the best designed mechanism will be better than the simple auction. In addition, if one extends the model to allow for common values, the result no longer holds. Bulow and Klemperer (2009) compare a standard English auction to a negotiation that is defined as a sequential procedure, where in each round a new bidder might enter the negotiation, and then competes head on with any bidder left from previous rounds. In case he wins this competition, he can make a jump bid in order to deter further entry. Bulow and Klemperer (2009) show that in this context, the auction fares better in terms of revenue although the negotiation is more efficient. This is due to the fact that entrants have to incur costs to learn their true valuation. Thus, bidders may prevent further entry with pre-emptive bids thereby capturing most of the efficiency gains. However, Davis et al. (2013) find in an experiment that in the same setting the negotiation outperforms the auction as subjects enter the negotiation more often than the auction and fail to employ the optimal pre-emptive bids.
In our set-up, the negotiation also is more likely to be the efficient mechanism: the gain in efficiency is due to the fact that the negotiation is less likely to be manipulated and the optimal specification for the buyer is more likely to be implemented. Hence, contrary to Bulow and Klemperer (2009), the buyer is able to capture most of the efficiency gain and thus may benefit from the negotiation.\footnote{Other approaches to the comparison of auctions and negotiations include McAdams and Schwarz (2006), Fluck et al. (2007) or Manelli and Vincent (1995).}

The major challenge in comparing auctions and negotiations is to find a precise definition for each of the mechanisms. The sparse literature on this subject uses different approaches to tackle this issue. We argue that one of the main differences between both formats is the timing at which the precise rules are set and show that, contrary to previous works, negotiations can outperform auctions.

2. The Model: Defining “Auction” and “Negotiation”

Suppose a buyer procures one indivisible project from $N$ risk neutral sellers. Let $i \in \{1, \ldots, N\}$ index the sellers. Each of the sellers has privately known costs $c_i$ of delivering the project. It is common knowledge that $c_i$ is distributed with c.d.f. $F$ on support $[0, \bar{c}]$. The sellers are horizontally differentiated with respect to the specifications of the project. This is captured for seller $i$ by a given location $q_i$ along the specification space $[\underline{q}, \bar{q}]$. For each $i$, $q_i$ is known to the buyer. If seller $i$ is selected to deliver the project at a price $p$ the value to the buyer is $V - |\theta - q_i| - p$ with $V \in \mathbb{R}_+$.\footnote{Assuming that the value to the buyer is $V(|\theta - \hat{\theta}|)$ for some concave function $V$ does not change our results qualitatively.} The parameter $\theta \in [\underline{q}, \bar{q}]$ represents the desired specification of the buyer and is not observed by the buyer prior to the procurement process.

The buyer has to delegate the execution of the procurement mechanism to an agent who can privately observe the parameter $\theta$ prior to procuring the project.\footnote{For example, we can think of the buyer being the public and the agent being a bureaucrat in charge of running a public procurement. In this case, it is easy to make sense of the assumption that the agent is better informed about the preferences of the buyer than the buyer himself. See Arozamena and Weinschelbaum (2009), Burguet and Perry (2007), Celentani and Ganuza (2002), or Laffont and Tirole (1991) for an exhaustive description of such situations.} The auctioneer colludes with one of the sellers and may favor this seller by misrepresenting $\theta$ by announcing some $\hat{\theta}$ to the buyer. In what follows, let seller 1 be the seller in question.\footnote{We assume that the favorite bidder is exogenously given. This assumption is a good approximation for many situations in public procurement where the agent may have a well established relationship with the domestic firm.} The agent maximizes the surplus of seller 1 and has a weak preference for honesty, i.e., whenever seller
1 cannot strictly benefit from manipulation of $\theta$ the agent prefers to announce $\theta = \hat{\theta}$. We define and compare two different procurement mechanisms – auctions and negotiations:

**Auction.** An auction is conducted under full public scrutiny, i.e., all relevant dimensions of the auction have to be made publicly available prior to its start. Hence, in an auction the agent has to set all relevant parameters and procedures of a specific auction format before the sellers submit their offers.\(^\text{11}\) Moreover, public scrutiny implies that even if the buyer is not aware of his preferred specification $\theta$, once the auction format has been set, auction experts can point out whether the proposed auction format is optimal given some feasible specification $\hat{\theta}$. Thus, in the context of public procurement, it is reasonable to assume that the agent has to implement the optimal auction given some $\hat{\theta} \in [q, \bar{q}]$.\(^\text{12}\)

The timing of the auction is the following:

- (i) The agent privately observes $\theta$.
- (ii) The agent publicly commits to the buyer surplus-optimal auction given some $\hat{\theta} \in [q, \bar{q}]$.
- (iii) The sellers submit bids to the agent and the winning bidder is determined.\(^\text{13}\)

**Negotiation.** The negotiation is conducted privately by the agent and the process cannot be publicly observed. Thus, in a negotiation the agent is not bound by the requirement to set all the relevant parameters and procedures in advance. He is rather free to choose his decision criteria at any time during the process. Even though the negotiation is conducted privately, the agent has to publicly rationalize his final decision. Hence, some public scrutiny cannot be avoided. Public scrutiny places two restrictions on the decision of the agent.

First, the agent cannot prevent any of the bidders from submitting offers. This is due to the fact that in public procurement the contracting authority has “obligations regarding information [...]”. This takes the form of publishing information notices [...] prior to the

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\(^{\text{11}}\)The public procurement directive of the European Union states concerning (electronic) auctions: “The electronic auction shall be based [...] on prices and/or values of the features of the tenders, when the contract is awarded to the most economically advantageous tender. The specifications shall contain [...] the quantifiable features (figures and percentages) whose values are the subject of the electronic auction and the minimum differences when bidding. [...] The invitation shall state the mathematical formula to be used to determine automatic rankings, incorporating the weighting of all the award criteria.” (See the “Directive 2004/18/EC of the European Parliament and of the Council of 31 March 2004 on the coordination of procedures for the award of public works contracts, public supply contracts and public service contracts”).

\(^{\text{12}}\)In a related project, we investigate in how far public scrutiny constrains the agent to rig the rules of the auction. For the purpose of the present paper, assuming that public scrutiny forces the agent to use the optimal auction is sufficient. Note that allowing the agent to implement an auction of his choice will reinforce our results in favor of the negotiation.

\(^{\text{13}}\)Bidders are committed to their offers.
start of the procurement process.\footnote{See the above mentioned “Directive 2004/18/EC” on public procurement.} Hence, all relevant sellers are aware that the project is being procured and could appeal against the exclusion of their offers.

Second, the agent has the obligation to reveal the winner of the process and the final agreement to the buyer. Moreover, the sellers that did not win the project may request a statement by which means their offer is inferior to the offer of the winner.\footnote{For example, the public procurement directive of the European Union states: “Each contracting authority shall provide information, as soon as possible, on the decisions reached concerning the award of a contract, including grounds for not awarding it. [...] On the request of the economic operator concerned [the contacting authority should provide information on] any unsuccessful candidate of the reasons for rejecting them; any tenderer who has made an admissible tender of the relative advantages of the tender selected, as well as the name of the economic operator chosen.” (See the above mentioned “Directive 2004/18/EC” on public procurement).} In our set-up the specification and the price that a seller receives are the only relevant decision dimensions. Hence, this kind of public scrutiny places a restriction on the decision of the agent in the sense that the final winning offer has to be the lowest of all submitted offers for the implemented specification. Otherwise, it is clear that the negotiation has been manipulated and the agent is punished with a sufficiently large fine.\footnote{Observe that if this restriction is relaxed, the comparison of auctions and negotiations becomes meaningless, as in the negotiation the agent could simply give the project to his favorite bidder at price $V$ and discard all the other offers. A similar argument applies if the agent is not obligated to take at least one offer from each seller as in the first restriction. Hence, the obligation to take at least one offer from each seller and to award the project to the seller with the lowest offer at the implemented specification are in a sense minimal.}

These two requirements place only little restriction on how the agent conducts the negotiation, in particular on how the agent may come to a final decision respecting the public scrutiny requirements. We explore the two fundamental ways for the agent to conduct the negotiation: he can reject offers or he can accept offers. Rejecting offers implies that the agent can credibly tell a seller that his current offer does not suffice to win the project. A seller whose offer has been rejected may then resubmit a better offer. If the seller does not resubmit an offer, the agent can exclude him from the further process. If all offers but one have been rejected, this offer is the winning offer. This case is analyzed below. In contrast, accepting offers implies that the agent can credibly declare one offer as the winning offer and award the project to the respective seller without taking any further offers. This is subject of Section 6.

If the agent can credibly reject offers, the negotiation takes the following form:

(i) The agent privately observes $\theta$.

(ii) In each round $t \ (t \in \mathbb{N})$ of the negotiation, each honest seller $i \in \{2,\ldots,N\}$ may submit an offer $p^t_i$ to the agent.\footnote{Sellers are committed to their offers.}
(iii) The agent observes the offers and shows them to seller 1.
(iv) Seller 1 may submit an offer $p_1^t$ or leave the negotiation.
(v) The agent rejects one or more offers of the sellers.
(vi) A bidder whose offer was rejected may submit an improved offer, i.e., $p_{t+1}^i < p_t^i$. If he submits a new offer (iii) to (vi) are repeated. If he does not submit an improved offer, the agent can exclude him from the further process.
(vii) The negotiation ends if all but one bidder were excluded from or left the negotiation.

This bidder is declared the winning bidder. The agent sets the final specification $\hat{\theta} \in [q, \bar{q}]$. The winning bidder is paid $p_t^i$.

Public scrutiny implies that if bidder $i$ is the winning bidder in Round $\bar{t}$, $V - |\hat{\theta} - q_i| - p_t^i \geq \max_{j \neq i} \left( V - |\hat{\theta} - q_j| - \min_{t \leq t} p_t^j \right)$ has to hold.\(^\text{18}\) To illustrate the public scrutiny requirement suppose that there are two offers on the table—the offer of bidder $j$ is $p_j$ while bidder $i$ makes an offer $p_i$ as depicted in Figure 1. As argued above, at the end of the process, the final agreement and $\hat{\theta}$ have to be revealed to the buyer and the loosing sellers. If the agent announces $\hat{\theta}$ as the buyer’s preferred specification, he can claim that bidder $j$ has the lowest offer. If the offers are as depicted in Figure 2, there is no announcement

\(^{18}\)To fully characterize the game, we assume that if the agent rejects all offers, or he violates public scrutiny, the agent pays a sufficiently large fine $D$. 

**Figure 1.** With the appropriate choice of $\hat{\theta}$ the agent can declare bidder $j$ as the winning bidder.
of \( \hat{\theta} \) such that the agent can claim that bidder \( j \) has the lowest offer without violating the public scrutiny constraint.

3. Equilibrium Allocations in the Mechanisms

In this section, we derive the equilibria for the auction and the negotiation.

3.1. Equilibrium allocation in the auction. First, we derive the buyer surplus-optimal auction for a given specification \( \hat{\theta} \). To simplify the exposition, we make a standard assumption that ensures that it is always optimal to procure the object:

**Assumption 1.** The following holds true for all \( c \in [c, \bar{c}] \):

(i) \( V - |q - \theta| - c - F(c)/f(c) \geq 0 \) for all \( q, \theta \in [q, \bar{q}] \)

(ii) \( \psi(c) := c + F(c)/f(c) \) is strictly increasing in \( c \).

Assumption 1 is satisfied if \( F(c)/f(c) \) is non-decreasing and \( V \) is sufficiently large. We use the revelation principle and restrict our attention to direct revelation mechanisms: \( g_i(c) \) denotes the awarding rule - the probability of winning the project for firm \( i \); \( t_i(c) \) denotes the expected payment to firm \( i \) if the vector of announced costs is \( c = (c_1, \ldots, c_N) \).\(^{19}\) The optimal auction can be described as follows:

\(^{19}\)The specification \( q_i \) is known to the buyer. Hence, it suffices to restrict our attention to direct mechanisms that ask the sellers to report their cost \( c_i \).
Lemma 1. Suppose Assumption 1 holds true. The optimal auction for a given $\hat{\theta}$ is fully characterized by the awarding rule:

\[
g^\hat{\theta}(c) = \begin{cases} 
1 & \text{if } V - c - |q - \hat{\theta}| - \frac{F(c)}{f(c)} > V - c_j - |q_j - \hat{\theta}| - \frac{F(c_j)}{f(c_j)} \forall j \neq i \\
0 & \text{otherwise.}
\end{cases}
\]

The expected surplus of seller $i$ is given by

\[
U_i(\hat{\theta}, c_i) = \int_{c_i}^{\hat{\theta}} \int g^\hat{\theta}(s, c_{-i}) dF^{N-1}(c_{-i}) ds.
\]

The expected profit of the buyer in terms of his true specification $\theta$ is given by

\[
\Pi_a(N) := E_c \left[ \sum_{i=1}^{N} g^\hat{\theta}(c) \left( V - |\theta - q_i| - c_i - \frac{F(c_i)}{f(c_i)} \right) \right].
\]

The expected social surplus in terms of the true specification $\theta$ is given by

\[
\xi_a(N) := E_c \left[ \sum_{i=1}^{N} g^\hat{\theta}(c) \left( V - |\theta - q_i| - c_i \right) \right].
\]


Sellers with a specification $q_i$ that is close to $\hat{\theta}$ have a relative advantage. If all sellers are treated equally, those sellers would bid less aggressively and thereby lower the buyer surplus. Hence, the optimal awarding rule discriminates against those sellers and thereby elicits more aggressive bidding.\(^{20}\)

The optimal auction can be implemented as a first- or second-score auction.\(^ {21}\) Hence, it is meaningful to speak about auctions in the context of this paper. We are only interested in the resulting buyer and seller surplus. Thus, we will refrain from deriving the exact scoring rules and just state the following lemma:

Lemma 2. Let $b^f_i$ denote the bid of firm $i$ in a first-score auction and $b^s_i$ the bid of firm $i$ in a second-score auction. There exist scoring rules $W^f(q_i, b^f_i)$ for the first-score auction and $W^s(q_i, b^s_i)$ for the second-score auction such that in equilibrium the buyer and seller surplus coincides with the surplus in the optimal auction.

\(^{20}\)To illustrate this discrimination, suppose that $F(c) = c$ and $N = 2$. In the optimal auction, seller 1 wins whenever $2c_1 + |q_1 - \hat{\theta}| < 2c_2 + |q_2 - \hat{\theta}|$, where as in an efficient mechanism seller 1 wins whenever $c_1 + |q_1 - \hat{\theta}| < c_2 + |q_2 - \hat{\theta}|$. Thus, the specification advantage has less weight than the cost advantage.

\(^{21}\)In a first-score auction, each seller transmits a bid $b^f_i$. The seller with the highest score $W^f(q_i, b^f_i)$ is selected as a winner and receives a payment equal to his bid. In a second-score auction, each seller transmits a bid $b^s_i$. The seller with the highest score $W^s(q_i, b^s_j)$ is selected as a winner and receives a payment $p^*$ such that $W^s(q_i, p^*) = W^s(q_j, b^s_j)$ where $j$ is the bidder with the highest rejected score.

To fully characterize the auction mechanisms it remains to find $\hat{\theta}$ that maximizes the expected utility of seller 1. From expressions (1) and (2) it follows that maximizing the expected utility is equivalent to maximizing the winning probability of seller 1. The winning probability of seller 1 is maximized for

$$q_1 = \arg\max_{\hat{\theta}} \text{Prob}_{c_i} \left[ V - c_1 - |q_1 - \hat{\theta}| - F(c_1)/f(c_1) \geq \max_{i \neq 1} V - c_i - |q_i - \hat{\theta}| - F(c_i)/f(c_i) \right].$$

However, if $|q_1 - q_i| = |q_i - \theta| - |q_1 - \theta|$ for all $i \neq 1$, $\theta$ is also a maximizer of (5). As the agent has a weak preference for honesty, he in this case prefers to set $\hat{\theta} = \theta$. We summarize this finding in the following:

**Corollary 1.** In the auction the agent will set $\hat{\theta} = q_1$ if there exist $i \neq 1$ with $|q_1 - q_i| > |q_i - \theta| - |q_1 - \theta|$. The agent will set $\hat{\theta} = \theta$ otherwise.

### 3.2. Equilibrium allocation in the negotiation

We start the analysis of the negotiation by characterizing the behavior of the honest sellers and the equilibrium outcome.

While we do not put any constraint on how the negotiation is conducted, the two public scrutiny requirements and the assumption that the agent can credibly commit to reject offers, allows us to derive the allocation of the negotiation. This is done by deriving necessary properties of equilibrium allocations for any negotiation protocol that is consistent with the description of the negotiation stated in Section 2. In Appendix A we define and solve a specific negotiation game to illustrate the results. We proceed in four steps:

1. **For all honest sellers it is dominated not to lower their offers as long as their offers are above marginal cost and get rejected.** To see this observe that a honest seller whose offer was rejected has no chance to win the project if he does not make a new, lower offer. As long as $p^i_t > c_i$, by submitting a lower offer, the seller receives an expected surplus of at least zero.\(^{22}\) If, contrary to that, $p^i_t < c_i$, the seller receives the project, the surplus of this seller will be negative. Hence, if $p^i_t$ has been rejected and $p^i_t > c_i$, not submitting a new offer is weakly dominated by lowering $p^i_t$. Similarly, if $p^i_t = c_i$, lowering $p^i_t$ is weakly dominated by not submitting a new offer. Thus, for all honest sellers it is optimal to lower their offers if it becomes rejected until their offer is equal to the cost of delivering the project.

\(^{22}\)The surplus is strictly positive if the negotiation stops at a price $p^i_t > c_i$. 
(ii) For any final $\hat{\theta}$, the project is awarded to the seller $i$ whose cost and quality parameter maximize $V - |\hat{\theta} - q_i| - c_i$. From (i) it follows that if bidder $j \neq 1$ exists, he exits at prices equal to his costs. Similarly, bidder 1 would only exit if he would need to bid a price lower than his costs. Now, public scrutiny implies that in order for seller $j$ to win $V - |\hat{\theta} - q_j| - c_j \geq \max_{j \neq i} (V - |\hat{\theta} - q_j| - c_j)$. As winning is only favorable if $p_{\bar{t}} \geq c_i$ the cost and quality parameter of the winning seller must maximize $V - |\hat{\theta} - q_i| - c_i$. 

We summarize (i) and (ii) in the following proposition.

**Proposition 1.** In any equilibrium of the negotiation in undominated strategies each bidder $i$ will resubmit a new, lower offer if his offer is rejected or leave the negotiation if $p_i = c_i$. Thus, for any final $\hat{\theta} \in [q, \bar{q}]$, seller $j$ wins the project iff $V - |\hat{\theta} - q_i| - c_i \geq \max_{j \neq i} (V - |\hat{\theta} - q_j| - c_j)$.

Hence, any undominated equilibrium of the negotiation is efficient in the following sense: Given a final $\hat{\theta}$, the negotiation selects the seller who maximizes the overall surplus at specification $\hat{\theta}$. However, $\hat{\theta}$ might be chosen inefficiently by the agent.

(iii) The agent will set $\hat{\theta} = \theta$ whenever seller 1 fails to win. The agent has two objectives when maximizing the joint surplus. First, seller 1 should receive the project whenever his offer is the lowest offer of the other sellers at some specification $\hat{\theta}$. Second, whenever seller 1 fails to win the project the agent has a weak preference for honesty and prefers to set the true specification. As we have shown above (Proposition 1), the honest bidders will lower their offers to marginal costs if their offers are rejected. Hence, whether seller 1 can underbid the lowest offer of the honest sellers and receive the project is independent of the rejection strategy of the agent. Thus, it comes without cost to reject offers of honest bidders based on the true specification $\theta$, i.e., reject all offers but the offer $p_i$ that maximizes $V - |\theta - q_i| - p_i$. In addition, not rejecting the lowest offer on the true specification has the advantage that whenever the agent realizes that seller 1 cannot profitably win the project, he awards the project to the seller whose offer maximizes the surplus of the buyer for his true specification.

(iv) The agent will set $\hat{\theta} = \{q_1, \theta\}$ if seller 1 wins the project. This follows directly from what have been said before: Seller 1 will win the project if he can underbid all at some $\hat{\theta}$. Seller 1, as he can always observe all offers, then receives a price $p_1^\text{bar}$ such
that \( p_t^1 + |q_1 - \hat{\theta}| = \min_{i \neq 1} p_t^i + |q_i - \hat{\theta}| \). This is maximized for \( \hat{\theta} = q_1 \). However, if \(|q_i - \theta| - |q_1 - \theta| - |q_1 - q_i|\), the highest final price seller 1 can receive is the same for \( \hat{\theta} = q_1 \) and for \( \hat{\theta} = \theta \). Thus, in this case the agent prefers to set the true specification. The following proposition summarizes the equilibrium behavior of the agent.

**Proposition 2.** The following strategies maximize the surplus of seller 1 and the agent.

**Strategy of seller 1:**

(i) If \( c_1 \leq \min_{i \neq 2} p_t^i + |q_i - q_1|\), seller 1 bids \( p_t^1 = \min_{i \neq 2} p_t^i + |q_i - q_1| \) and stays in the negotiation.

(ii) Otherwise, seller 1 leaves the negotiation.

**Strategy of the agent:**

(i) If exactly one honest seller is active and seller 1 has submitted an offer the agent rejects the offer of the honest seller.\(^{23}\)

(ii) Otherwise, the agent rejects all offers but one of the offers \( j \in \arg \max_i V - |q_i - \theta| - p_t^i \).

If at the end of the process seller 1 is the last active seller and \(|q_i - \theta| - |q_1 - \theta| < |q_1 - q_i|\) for \( i \in \arg \min_{i \neq 1} p_t^i + |q_1 - q_i| \), the agent sets \( \hat{\theta} = q_1 \). Otherwise the agent sets \( \hat{\theta} = \theta \).

A more formal treatment of Proposition 2 for a specific negotiation game can be found in Appendix A. Combining Proposition 2 with Proposition 1 yields the equilibrium outcome of the negotiation in terms of an awarding rule of a direct revelation mechanism:

**Lemma 3.** The equilibrium outcome of the negotiation is equivalent to the outcome of a direct revelation mechanism characterized by the following awarding rule \( g^n(c) \):

\[
\begin{align*}
g^n_1(c) & = 1 \quad \text{if} \quad c_1 \leq \min_{j \neq 1} c_j + |q_j - q_1| \\
g^n_1(c) & = 0 \quad \text{otherwise}; \\
g^n_i(c) & = 1 \quad \text{if} \quad c_i + |q_i - \theta| \leq \min_{j \neq i} \{c_j + |q_j - \theta|\} \quad \text{and} \quad \min_{j \neq 1} \{c_j + |q_1 - q_j|\} < c_1, \quad i \neq 1 \\
g^n_i(c) & = 0 \quad \text{otherwise.}
\end{align*}
\]

The expected surplus of seller \( i \) is given by

\[
U^n_i(c_i) = \int_{c_i}^{\hat{c}} \int g^n_i(s, c_{-i}) dP^{N-1}(c_{-i}) ds.
\]

\(^{23}\)We call sellers active in round \( t \) that have not been excluded from the negotiation in previous rounds.
The expected profit of the buyer in terms of his true specification $\theta$ is given by

\[ \Pi_n(N) := E_c \left[ \sum_{i=1}^{N} g^n_i(c) \left( V - c_i - |q_i - \theta| - \frac{F(c_i)}{f(c_i)} \right) \right]. \]

The expected social surplus in terms of the true specification $\theta$ is given by

\[ \xi_n(N) := E_c \left[ \sum_{i=1}^{N} g^n_i(c) \left( V - c_i - |q_i - \theta| \right) \right]. \]

4. **Buyer Surplus**

To compare both mechanisms along the specification space, we assume that $q = (q_1, \ldots, q_N)$ is drawn from a continuous distribution $F_q$ on $[\underline{q}, \bar{q}]^N$ with $f_{q_i} > 0$ and assess the probability over $q$ that the buyer surplus from the auction mechanism ($\Pi_a(N)$) exceeds the buyer surplus from the negotiation ($\Pi_n(N)$). We will show that if $N = 2$ the auction always yields a higher surplus than the negotiation. However, there exists a lower bound on the number of sellers such that the probability that the auction generates more buyer surplus than the negotiation becomes arbitrarily small (smaller than any $\delta \in (0, 1)$).

As noted in Section 3, the optimal auction discriminates against sellers with a specification close to $\hat{\theta}$. The negotiation, however, selects the seller who maximizes the total surplus for $\hat{\theta}$ but leaves him with more rent. Whenever both mechanisms are manipulated, manipulation gives seller 1 an advantage by moving $\hat{\theta}$ to his specification $q_1$. Because of the mentioned discrimination, this advantage is less valuable in the auction. No such discrimination takes place in the negotiation, and seller 1 can fully benefit from the manipulation. Hence, the auction generates a higher buyer surplus if both mechanisms are manipulated.\footnote{Similarly, if no mechanism is manipulated, the auction generates a higher buyer surplus.} However, the negotiation is not always manipulated. This is due to the fact that in the negotiation, the agent observes the offers of the other sellers before choosing the final $\hat{\theta}$. Whenever the realization of $c_1$ is such that seller 1 cannot benefit from manipulation ex-post, the agent chooses not to manipulate the preferred specification.

If the number of seller increases two effects are relevant. First, the probability that the agent manipulates the negotiation approaches zero. Second, the rent that each seller receives from either mechanism decreases. Both effects favor the negotiation. Hence, the negotiation becomes more profitable.

**Proposition 3.** The following holds true:
(i) If $N = 2$ the auction yields a higher buyer surplus than the negotiation, i.e.,
$\Pi_a(N) \geq \Pi_n(N)$.

(ii) For every $\delta \in (0, 1)$ there exists $\tilde{N}(\delta) > 2$ such that the negotiation yields a higher buyer surplus with probability $1 - \delta$ for all $N > \tilde{N}(\delta)$, i.e.,
$\text{Prob}_q(\Pi_n(N) > \Pi_a(N)) > 1 - \delta$.

Proof. The proof is relegated to the appendix. \hfill \square

Proposition 3 is inconclusive about the ranking of the buyer surplus of both mechanisms if $N$ is larger than 2 but below $\tilde{N}$. The following example illustrates that for intermediate $N$, the buyer surplus can be higher in each of the formats with positive probability depending on $q$.

**Example 1.** Let $N = 3$, $V = 2$, $\theta = 0.5$, $c \sim U[0, 1]$, $q_2 = 0$, $q_3 = 1$, and $q_1 \in [0, 1]$. The expected surplus of the buyer in the auction and negotiation can be calculated using expressions (3) and (6). Figure 3 illustrates that buyer surplus can be larger in the auction or the negotiation depending on $q_1$. Applying the terminology of Proposition 3 it follows that if $q_1$ is distributed with a continuous distribution function $F_{q_1}$ with full support on $[\underline{q}, \overline{q}]$, $0 < \text{Prob}_{q}(\Pi_a(3) > \Pi_n(3)) < 1$ holds. Moreover, depending on $F_{q_1}$, $\text{Prob}_{q_1}(\Pi_a(3) > \Pi_n(3))$ can be arbitrary close to zero or one.
5. Efficiency

In this section, we will show that if the number of sellers is sufficiently large the negotiation is more efficient with probability close to one.

For the comparison of efficiency of both formats three cases are relevant: (i) Both mechanisms are manipulated, (ii) the auction is manipulated but not the negotiation, and (iii) both mechanisms are not manipulated. However, if the expected punishment is sufficiently low, the auction is always manipulated and the third and the fourth case are not relevant for our comparison.\(^{25}\)

If both, the auction and the negotiation, are manipulated (case (i)), the allocation in the auction is less distorted towards seller 1 but more distorted for the other sellers. This is due to the fact that the optimal auction discriminates against sellers with a specification close to \(\hat{\theta}\). Hence, in this case, either mechanism can be more efficient depending on the actual \((q, \theta)\) parameters. If the negotiation is not manipulated (cases (ii) and (iii)), the negotiation is the fully efficient mechanism and thus more efficient than the auction. If the number of sellers increases manipulation in the negotiation becomes less likely and case (i) less relevant. It follows that the negotiation is more efficient than the auction.

**Proposition 4.** For every \(\delta' \in (0, 1)\) there exists \(\bar{N}(\delta)\) such that the negotiation results in higher efficiency with probability \(1 - \delta\) for all \(N > \bar{N}(\delta)\), i.e., \(\text{Prob}_q[\xi_n(N) > \xi_a(N)] > 1 - \delta\).

**Proof.** The proof is relegated to the appendix. \(\Box\)

Proposition 4 is inconclusive about the ranking of the efficiency of both mechanisms if \(N\) is rather small. The following example illustrates that for small \(N\), the efficiency can be higher in each of the formats with positive probability depending on \(q\).

**Example 2.** Let \(N = 2, V = 2, \theta = 0.5\) and \(c \sim U[0, 1]\), \(q_1 \in [0, 1]\), and \(q_2 = 0\). The efficiency in the auction and negotiation can be calculated using expressions (4) and (7). Figure 4 illustrates that efficiency can be larger in the auction or the negotiation depending on \(q_1\). It follows that if \(q_1\) is distributed with a continuous distribution function \(F_{q_1}\) with full support on \([\tilde{q}, \bar{q}]\), \(0 < \text{Prob}_{q_1}[\xi_a(3) > \xi_n(3)] < 1\) holds. Moreover, depending on \(F_{q_1}\), \(\text{Prob}_{q_1}[\xi_a(3) > \xi_n(3)]\) can be arbitrary close to zero or one.

As \(\bar{N}\) is not necessarily smaller than \(\bar{N}'\), there exist parameter values such that the auction generates a higher buyer surplus but the negotiation is more efficient. From this

\(^{25}\)Nevertheless, if both mechanism are not manipulated the negotiation is the more efficient mechanism.
it directly follows that there exist parameter values such that the sellers receive a higher surplus in the negotiation. However, most of this surplus is captured by seller 1. The following proposition demonstrates that seller 1 prefers the negotiation over the auction.

**Corollary 2.** Seller 1 prefers the negotiation over the auction.

*Proof.* The proof is relegated to the appendix. □

Whether the honest sellers appropriate a higher surplus is uncertain. Most of the additional surplus that is captured by seller 1 in the negotiation is due to the fact that the negotiation does not discriminate against sellers with a favorable specification \( q_i \). Hence, he is able to capture all of the additional surplus from the manipulation in the negotiation. Whether the honest sellers prefer the negotiation over the auction depends therefore on how close their specification \( q_i \) is to the specification \( q_1 \) of seller 1.

### 6. Robustness

In deriving the negotiation procedure in Section 2 we have assumed that the agent can credibly *reject* the offers of the sellers. In this section we will focus on the case where the agent can credibly *accept* offers. Thus, we modify the negotiation procedure from Section
2 by allowing the agent to award the project to one of the sellers after collecting at least one offer from each seller. As the agent – to benefit his preferred seller – always prefers higher offers to lower offers, he will never inform one of the honest sellers before the end of the process whether his first offer was sufficient to win the project and thereby give him no chance to improve his offer. Hence, essentially, if the agent can credibly accept offers each seller submits exactly one offer and the negotiation takes the following form:

(i) The agent privately observes $\theta$.

(ii) Sellers submit an offer an offer $p_i$ to the agent.

(iii) The agent observes the offers and shows them to seller 1.

(iv) Seller 1 submit an offer $p_1$.

(v) The agent chooses the winning bidder and sets the final specification $\hat{\theta}$.

(vi) Public scrutiny implies that if bidder $i$ is the winning bidder, $V - |\hat{\theta} - q_i| - p_i \geq \max_{j \neq i} (V - |\hat{\theta} - q_j| - p_j)$ has to hold.

(vii) The winning bidder is paid $p_i$.

The strategy that maximizes joint surplus of the agent and seller 1 is straightforward:

(i) If there exist $i \neq 1$ with $|q_1 - q_i| > |q_i - \theta| - |q_1 - \theta|$, the agent sets $\hat{\theta} = q_1$ and seller 1 offers $p_1 = \max\{\min_{j \neq 1} p_j, c_1\}$.

(ii) If $t |q_1 - q_i| = |q_i - \theta| - |q_1 - \theta|$ for all $i \neq 1$, the agent sets $\hat{\theta} = \theta$ and seller 1 offers $p_1 = \max\{\min_{j \neq 1} p_j, c_1\}$.

For the honest bidders, the problem of choosing an optimal offer is essentially the same as choosing a bid in a asymmetric first-price auction with a stochastic reserve price. An equilibrium for this game is known to exist. However, a closed-form solution for the bidding strategies is hard to derive.

Nevertheless, due to the fact that in equilibrium $p_i > c_i$ and $\lim_{N \to \infty} p_i = c_i$ has to hold for all $i \neq 1$, the buyer surplus result from Section 4 also holds for the negotiation at hand: if $N$ is sufficiently small, the auction and the negotiation are both manipulated with a high probability. Manipulation then gives seller 1 a specification advantage over the other sellers. However, this advantage is less valuable in the auction as it discriminates against sellers with such an advantage. The allocation in the auction is less distorted than in the negotiation in which seller 1 can fully benefit from the manipulation. Hence, the auction with favoritism may generate a higher buyer surplus for small $N$. However, if the number of sellers grows, the outcome of the negotiation converges to the outcome

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26 The bid of the corrupt seller 1 resembles a stochastic reserve price.

27 See Athey (2001).
characterized in Section 3 as $\lim_{N \to \infty} p_i = c_i$. In this case, we know from Proposition 3 that the buyer surplus from the negotiation exceeds the buyer surplus from the auction with high probability. Hence, the negotiation generates a higher buyer surplus than the auction if $N$ grows. We summarize this finding in the following:

**Corollary 3.** The negotiation generates a higher buyer surplus than the auction if $N$ is sufficiently large.

7. Conclusion

We have shown that – contrary to common wisdom – the transparency of an auction does not render it favoritism proof. If the agent of the buyer is able to manipulate the specification of the procured project, an intransparent negotiation may be more efficient and generate more buyer surplus. This is due to the fact that in the auction, public scrutiny forces the agent to decide whether to manipulate the process before sellers submit their offers. In the negotiation on the other hand, after observing the offers of the sellers, the agent may still decide not to manipulate if he realizes that his preferred seller is not able to win the project.

If the specification is manipulated in both procedures, the auction is the optimal mechanism that implements the manipulated specification. In those cases, the auction will outperform the negotiation. However, if the auction is manipulated but not the negotiation, the negotiation may generate more surplus. This difference in manipulation is due to the fact that the auction is always manipulated. The negotiation, on the other hand, may not be manipulated because after observing the offers of the honest sellers, the agent may realize that his preferred seller has no chance of winning the project. This becomes more likely if the number of sellers increases.

This paper sheds light on the question whether auctions or negotiations should be used when designing a public procurement mechanism. We have argued that a seemingly straightforward reasoning that auctions – because of their transparency – should be preferred in the presence of favoritism does not apply. Whether an auction should be used over a negotiation depends on the number of participating sellers and the attributes of the sellers.
In this section we define and solve a specific negotiation game that illustrates our results from Section 3.

Define a price grid \( P = \{c, c + \Delta, \ldots, c + k\Delta \} \) with \( \Delta = \frac{c - \bar{c}}{k} \) for some \( k \in \mathbb{N} \). Define by \( A_t \subset \{1, \ldots, N\} \) the set of bidders that are still active at round \( t \) and set \( A_1 = \{1, \ldots, N\} \). Denote by \( p^t := (p^t_1, \ldots, p^t_N) \) the vector of prices offered by the bidders. The negotiation game can then be described as follows

**Round 1.**

(i) Each seller \( i \in A_1 \setminus \{1\} \) submits a price \( p^1_i \in P \) to the agent

(ii) The agent and seller 1 observe all offers \( p^1_i \) and seller 1 offers \( p^1_1 \)

(iii) The agent informs each seller \( i \) whether his offer has been rejected. This is captured in a vector \( r^1 \) with \( r^1_i = 1 \) if the offer of seller \( i \) was rejected and \( r^1_i = 0 \) if the offer of seller \( i \) was not rejected.

**Round \( t \).**

(i) Each seller \( i \in A_t \setminus \{1\} \) submits a price \( p^t_i \in P \) to the agent subject to \( p^t_i \leq p^{t-1}_i \).

If \( p^t_i = p^{t-1}_i \) and \( r^{t-1}_i = 1 \) then seller \( i \) is removed from \( A_{t+1} \). For each seller \( i \notin A_1 \{1\} \) set \( p^t_i = p^{t-1}_i \)

(ii) The agent and seller 1 observe all the offers \( p^t_i \) and seller 1 offers \( p^t_1 \in P \) or leaves the auction. This is captured in \( r^t_1 \) with \( r^t_1 = 1 \) if seller 1 leaves the auction and \( r^t_1 = 0 \) otherwise. If seller 1 leaves the auction he is removed from \( A_{t+1} \)

(iii) The agent informs each seller \( i \in A_t \{1\} \) whether his offer has been rejected. This is captured in a vector \( r^t \) with \( r^t_i = 1 \) if the offer of seller \( i \) was rejected and \( r^t_i = 0 \) if the offer of seller \( i \) was not rejected.

The game ends in round \( \tau \) if \( |A_\tau| = 1 \). In this case the last active bidder is declared the winning bidder and is paid \( p^\tau_i = p^{\tau-1}_i \). The agent sets the final specification \( \hat{\theta} \in [q, \bar{q}] \). For convenience we will sometimes denote the final round as round \( f \). Public scrutiny implies that if bidder \( i \) is the winning bidder, \( |q_i - \hat{\theta}| + p^f_i \leq \min_{j \neq i} |q_j - \hat{\theta}| + p^f_j \) has to hold. If the public scrutiny constraint is violated, a sufficiently large fine is imposed on the agent. Thus, violation of the public scrutiny constraint can never be part of an equilibrium of the game.

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\(^{28}\)In our description of the negotiation in Section 3 we leave it to the agent to remove sellers from the set of active bidders if they are rejected and don’t lower their offer. Here the sellers are removed from the set of active bidders as part of the description of the game. This greatly simplifies the exposition of the strategy of the agent. This is without loss of generality as removing honest sellers from the set of active bidders whenever possible is clearly maximizes the surplus of seller 1 and relaxes the public scrutiny constraints.
To express equilibrium strategies of the agent and the bidders some definitions are in order. A history $h^t$ at stage $t$ is defined by

$$h^t := (p^1, \ldots, p^{t-1}, r^1, \ldots, r^{t-1}, A_1, \ldots, A_{t-1}).$$

The agent and seller 1 always observe the whole history at stage $t$. Any honest seller $i \in 2, \ldots, N$ only observes his private history $h^t_i := (p^1_i, \ldots, p^{t-1}_i, r^1_i, \ldots, r^{t-1}_i)$ and whether $i \in A_t$ which can be deducted from $h^t_i$. A strategy of the agent is a mapping with $\sigma_a(h^t) = r^t$ and $\sigma_a(h^f) = \hat{\theta}$. A strategy of a honest seller $i \in \{2, \ldots, N\}$ is a mapping with $\sigma_i(c, h^t_i) = p^t_i$. A strategy of seller 1 is a mapping $\sigma_1(c, h^t) = (p^t_1, r_1)$. Denote by

$$u_i(\sigma_a, \sigma_1, \ldots, \sigma_N) = \begin{cases} c_i - p^t_i & \text{if } i \in A_f \\ 0 & \text{otherwise} \end{cases}$$

the surplus of seller $i$. The surplus of the agent is identical with the surplus of agent 1 with the exception that the agent weakly prefers not to manipulate the auction if manipulation does not benefit seller 1. In what follows we describe an equilibrium of the previously defined game. In this equilibrium the honest sellers lower their offer by one price step if it is rejected. Honest sellers leave the auction as soon as lowering their offer one more time would result in a price below their marginal costs. Seller 1 observes all offers and submit the highest possible price that allows him to win at some specification as long as this price is above his costs. As long as more than one honest seller is active or if seller 1 has exited the negotiation, the agent rejects the offers of the honest sellers based on the true preference of the buyer. If only one honest seller and seller 1 are active in the negotiation, the agent rejects the offer of the honest seller.

Claim 1. The following strategies form an equilibrium of the negotiation game:

(i) Honest sellers: $\sigma^*_i(h^0_i) = \bar{c}$

$$\sigma^*_i(h^t_i) = \begin{cases} p^t_i - \Delta & \text{if } r^t_i = 0 \text{ or } p^t_i - \Delta < c_i \\ p^t_i - \Delta & \text{if } r^t_i = 1 \text{ and } p^t_i - \Delta \geq c_i \end{cases}$$

(ii) Seller 1

$$\sigma^*_1(h^t) = \begin{cases} (\min_{i \neq 1} p^t_i + |q_1 - q_i|, 0) & \text{if } \min_{i \neq 1} p^t_i + |q_1 - q_i| \geq c_1 \\ (c_1, 1) & \text{otherwise} \end{cases}$$
(iii) Agent

\[
\sigma^*_a(h^t) = \begin{cases} 
    r^t_i = 0 & \text{if } i = \min_j \arg \min_j p^t_j + |q_j - \theta| \text{ and } |A^t \setminus \{1\}| \geq 2 \\
    r^t_i = 0 & \text{if } i = \min_j \arg \min_j p^t_j + |q_j - \theta| \text{ and } r^t_1 = 1 \\
    r^t_i = 1 & \text{otherwise}
\end{cases},
\]

with \( \sigma^*_a(h^f) = q_1 \) if \( A^f = \{1\} \) and \( |q_i - q_1| > |q_i - \theta| - |q_1 - \theta| \) for \( i = \arg \min_{i \neq 1} p^t_i - |q_i - q_1| \), \( \sigma^*_a(h^f) = \theta \) otherwise.\(^{29}\)

**Proof.** We start with the strategies of the honest sellers. Consider some deviation \( \tilde{\sigma}_i \neq \sigma^*_i \).

If following \( \tilde{\sigma}_i \) implies that bidder \( i \) does not win the project, his surplus is 0 which is at least as good as the surplus from following \( \sigma^*_i \). Thus, suppose following \( \tilde{\sigma}_i \) results in winning the object. Observe, that with \( \sigma^*_i \) seller \( i \) always obtains the good whenever the final price is above \( c_i \). Hence, a profitable deviation \( \tilde{\sigma}_i \) cannot result in higher winning probability.

It follows that any profitable deviation must result in a higher final price at some histories. Suppose seller \( i \) wins the project. The strategy of the agent and the other sellers implies that for the final price \( p^t_i + |q_i - \theta| \leq \min_{j \neq i} c_j + |q_j - \theta| \) and \( p^t_i + |q_i - q_1| \leq c_1 \) has to hold. Moreover, following \( \sigma^*_i \) implies that \( p^t_i > \min \{ c_1 - |q_i - q_1|, \min_{j \neq i} c_j + |q_j - \theta| - |q_i - \theta| \} - \Delta \).

Thus, following \( \sigma^*_i \) yields the highest feasible price given the strategies of the agent and the other sellers that is on the price grid. Hence, following \( \tilde{\sigma}_i \) cannot constitute a strictly profitable deviation from \( \sigma^*_i \).

Next, we apply the single deviation principle and show that the strategy of seller 1 is optimal given the strategies of the other sellers and the agent. Without loss of generality we only consider histories in which seller 1 has not left the negotiation in previous rounds.

Case (i) \( h^t \) is such that \( \min_{i \neq 1} p^t_i + |q_1 - q_i| \geq c_1 \). If bidder 1 drops out his surplus from the negotiation is 0. Thus, we may consider only deviations from \( \sigma^* \) with \( r^t_1 = 0 \). Observe that if \( |A^t \setminus \{1\}| \geq 2 \), the price submitted by bidder 1 has no influence on the rejection strategy of the agent or the bidding strategy of the other sellers. Hence, we may restrict our attention to histories with \( |A^t \setminus \{1\}| = 1 \). Suppose seller 1 submits a price \( p \neq \min_{i \neq 1} p^t_i + |q_1 - q_i| \). If the last remaining honest seller drops out, this implies that either seller 1 gets paid less than when following \( \sigma^*_1 \) \((p < \min_{i \neq 1} p^t_i + |q_1 - q_i|)\) or that the final price violates the public scrutiny constraint \((p > \min_{i \neq 1} p^t_i + |q_1 - q_i|)\). If the last remaining bidder does not drop out, the price submitted in round \( t \) has no implications for the action set of round \( t + 1 \). Thus, bidding \( p \) is not a strictly profitable deviation from \( \sigma^*_1 \).

\(^{29}\)To simplify notation we abstract from the fact that \( \min_{i \neq 1} p^t_i + |q_1 - q_i| \) may be not element of the price grid. In this case we assume that seller 1 chooses the next lowest price on the price grid.
Case (ii) $h^t$ is such that $\min_{i \neq 1} p^t_i + |q_1 - q_i| < c_1$. Any deviation that involves $r_1^t = 1$ yields a surplus of 0 which is the same as the surplus from $\sigma^*$. Thus, suppose $r_1^t = 0$ and some bid $p$. If $p > \min_{i \neq 1} p^t_i + |q_1 - q_i|$, and seller 1 wins the project, the final price violates the public scrutiny constraints. If $p < \min_{i \neq 1} p^t_i + |q_1 - q_i|$, then $p < c_1$. Thus, if seller 1 wins the project, his surplus is negative. Hence, the proposed deviation from $\sigma^*_1$ is not profitable.

We turn our attention to the strategy of the agent. The strategy of the honest sellers $\sigma^*_i$ prescribes that seller $i$ will remain active as long as $p^t_i \geq c_i$. Thus, irrespective of the agents rejecting strategy, the surplus of seller 1 is at most $\min_{i} c_i + |q_1 - q_i| - c_1$. Together with the strategy of seller 1 $\sigma^*_a$ achieves that upper bound. Moreover, $\sigma^*_a$ is defined such that one of the sellers who have submitted the lowest offer at the true specification $\theta$ either wins the project or remains active until the last round. Thus, in case seller 1 does not win the project, the agent may set $\hat{\theta} = \theta$ without violating the public scrutiny constraints. Thus, there is no strategy that would make seller 1 and the agent better off given the strategies of seller 1 and the honest sellers. □

Appendix B. Proof of Proposition 3

Proof. ad (i): If $|q_2 - \theta| - |q_1 - \theta| = |q_1 - q_2|$ the auction is not manipulated and is the optimal mechanism. Thus, wlog we can suppose $|q_2 - \theta| - |q_1 - \theta| < |q_1 - q_2|$ and the auction is manipulated, i.e., $\hat{\theta} = q_1$. Without manipulation it is optimal that seller 1 receives the project whenever his virtual surplus $V - c_1 - |q_1 - \theta| - \frac{F(c_1)}{f(c_1)}$ is larger than the virtual surplus of seller 2. If the auction is manipulated, the allocation is distorted in favor of seller 1 and he receives the project in more cases than optimal. As by Assumption 1 the virtual surplus is decreasing in $c$ and there are only two sellers, it is sufficient to show that in the negotiation seller 1 receives the project in more cases than in the auction to prove $\Pi_a(N) \geq \Pi_n(N)$. Define $c^a_1$ as the lowest cost $c_1$ such that seller 1 receives the project in the auction given $c_2$, i.e.,

$$c^a_1 = c_2 + |q_1 - q_2| + \frac{F(c_2)}{f(c_2)} - \frac{F(c^a_1)}{f(c^a_1)},$$

and $c^n_1$ as the lowest cost $c_1$ such that seller 1 receives the project in the negotiation given $c_2$, i.e.,

$$c^n_1 = c_2 + |q_1 - q_2|.$$

As $c^a_1 > c_2$, $F(c_2)/f(c_2) < F(c^a_1)/f(c^a_1)$ by Assumption 1. Thus, $F(c_2)/f(c_2) - F(c^a_1)/f(c^a_1) < 0$ and $c^n_1 > c^a_1$. 

ad (ii) Observe that $\lim_{N \to \infty} \Pr_q \{ \exists i \neq 1 : |q_i - \theta| - |q_1 - \theta| < |q_1 - q_i| \} = 1$. Thus, the agent manipulates the auction and sets $\hat{\theta} = q_1$ with probability close to 1 if the number of bidders is high. If the auction is manipulated buyer surplus from the auction approaches $V - |q_1 - \theta|$ if the number of bidders is high. It follows that for every $\epsilon > 0$ there exists $N_1(\epsilon)$ such that

\begin{equation}
Pr_q \left[ -\epsilon \leq \Pi_a(N) - (V - |q_1 - \theta|) \leq \epsilon \right] > 1 - \delta
\end{equation}

for all $N > N_1(\epsilon)$.

The agent manipulates the negotiation if and only if $c_1 \leq \min_{i \neq 1} c_i + |q_i - q_1|$. Hence, the agent manipulates the negotiation with probability close to 0 if the number of bidders is high. If the negotiation is not manipulated buyer surplus from the negotiation approaches $V$. It follows that for every $\epsilon > 0$ there exists $N_2(\epsilon)$ such that

\begin{equation}
Pr_q \left[ -\epsilon \leq \Pi_n(N) - V \leq \epsilon \right] > 1 - \delta'
\end{equation}

for all $N > N_2(\epsilon)$. Comparing equation (8) and equation (9) for an appropriate choice of $\epsilon$ yields

$$Pr_q [\Pi_n(N) > \Pi_a(N)] > 1 - \delta$$

for all $N > \max \{N_1(\epsilon), N_2(\epsilon)\}$. Thus, setting $\bar{N} = \max \{N_1(\epsilon), N_2(\epsilon)\}$ yields the result. \qed

\textbf{Appendix C. Proof of Proposition 4}

\textit{Proof.} If the auction is manipulated efficiency from the auction approaches $V - |q_1 - \theta|$ if the number of bidders is high. It follows that for every $\epsilon > 0$ there exists $N_1(\epsilon)$ such that

\begin{equation}
Pr_q \left[ -\epsilon \leq \xi_a(N) - (V - |q_1 - \theta|) \leq \epsilon \right] > 1 - \delta'
\end{equation}

for all $N > N_1(\epsilon)$.

The agent manipulates the negotiation if and only if $c_1 \leq \min_{i \neq 1} c_i + |q_i - q_1|$. Hence, the agent manipulates the negotiation with probability close to 0 if the number of bidders is high. If the negotiation is not manipulated the efficiency from the negotiation approaches $V$ if the number of bidders is high. It follows that for every $\epsilon > 0$ there exists $N_2(\epsilon)$ such that

\begin{equation}
Pr_q \left[ -\epsilon \leq \xi_n(N) - V \leq \epsilon \right] > 1 - \delta'
\end{equation}
for all $N > N_2(\epsilon)$. Comparing equation (10) and equation (11) for an appropriate choice of $\epsilon$ yields

$$\text{Prob}_q [\xi_n(N) > \xi_a(N)] > 1 - \delta'$$

for all $N > \max \{N_1(\epsilon), N_2(\epsilon)\}$. Thus, setting $\bar{N}' = \max \{N_1(\epsilon), N_2(\epsilon)\}$ yields the result. □

**Appendix D. Proof of Corollary 2**

**Proof.** Lemma 1 can be used to write the expected utility of seller 1 as

(12) \[ U^n_1(q_1, c_1) := \int_{c_1}^{\bar{c}} \int g_i^n(c_1, c_{-1})dF^{N-1}(c_{-1})dc_1 \]

\[ = \int_{c_1}^{\bar{c}} \prod_{i=2}^{N} (1 - F(\psi^{-1}(-|q_i - q_1| + \psi(c_1))))dc_1. \]

Observe that

\[-|q_i - q_1| + c_1 < \psi^{-1}(-|q_i - q_1| + \psi(c_1))\]

\[\Leftrightarrow\psi(-|q_i - q_1| + c_1) < -|q_i - q_1| + \psi(c_1)\]

\[\Leftrightarrow -|q_i - q_1| + c_1 + \frac{F(-|q_i - q_1| + c_1)}{f(-|q_i - q_1| + c_1)} < -|q_i - q_1| + c_1 + \frac{F(c_1)}{f(c_1)}.\]

The last inequality is true as we assumed that $F(c_1)/f(c_1)$ is increasing.

The expected surplus of seller 1 in the negotiation can be written as

(13) \[ U^n_1(c_1) = \int_{c_1}^{\bar{c}} \int g_i^n(c_1, c_{-1})dF^{N-1}(c_{-1})dc_1 \]

\[ \geq \int_{c_1}^{\bar{c}} \prod_{i=2}^{N} (1 - F(-|q_i - q_1| + c_1))dc_1. \]

It follows that $U^n_1(q_1, c_1) \geq U^n_1(q_1, c_1)$. □

**References**


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