International Prices and Endogenous Quality*

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Abstract

The unit value of internationally traded goods are heavily influenced by quality. We model this in an extended monopolistic competition framework, where in addition to choosing price, firms simultaneously choose quality. We employ a demand system to model consumer demand whereby quality and quantity multiply each other in the utility function. In that case, the quality choice by firms’ is a simple cost-minimization sub-problem. We estimate this system using detailed bilateral trade data for over 150 countries for 1984-2008. Our system identifies quality-adjusted prices from which we will construct price indexes for imports and exports for each country, that will be incorporated into the next generation of the Penn World Table.

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1. Introduction

It has long been known that the unit value of internationally traded goods are heavily influenced by their quality (Kravis and Lipsey, 1974). Historically, that linkage was viewed in a negative light and is the reason why import and export prices indexes for the United States no longer use any unit-value information, but instead rely on price surveys from importers. More recently, it has been argued that the variation in unit values is systematically related to characteristics of the exporting (Schott, 2004) and importing (Hallak, 2006) countries. Such a relationship gives a positive interpretation to the linkage between unit values and quality because, as argued by Hummels and Klenow (2005) and Baldwin and Harrigan (2011), we can use this systematic variation to test between competing trade models.

Our goal in this paper is to estimate that portion of trade unit values that is due to quality. To achieve this we use the model identified by Baldwin and Harrigan (2011) as most consistent with the empirical observations – quality with heterogeneous firms – and extend it to allow for endogenous quality choice by firms.¹ We are not the first to attempt to disentangle quality from trade unit values, and other recent authors with that goal include Hallak and Schott (2011) and Khandelwal (2010).² These studies rely on the demand side to identify quality. In the words of Khandelwal (2010, p. 1451): “The procedure utilizes both unit value and quantity information to infer quality and has a straightforward intuition: conditional on price, imports with higher market shares are assigned higher quality.” Likewise, Hallak and Schott (2011) rely on trade balances to identify quality. To this demand-side information we will add a supply side, drawing on the well-...

¹ Other models with endogenous quality choice by heterogeneous firms include Gervias (2010), Khandelwal (2010) and Mandel (2009). The latter two paper have simultaneous choice of price and quality, as we use here. In contrast, Gervias has quality chosen for the lifetime of a product. This yields a solution where quality is proportional to firm productivity, thereby providing a micro-foundation for that assumption in Baldwin and Harrigan (2011).
known “Washington apples” effect (Alchian and Allen, 1964; Hummels and Skiba, 2004): goods of higher quality are shipped longer distances. We will find that this positive relationship between exporter f.o.b. prices and distance is an immediate implication of the first-order condition of firms for optimal quality choice. This first-order condition gives us powerful additional information from which to identify quality.

In section 2, we specify an extended monopolistic competition framework, where in addition to choosing price, firms in each country simultaneously chooses quality. Like the early work by Rodriguez (1979), we allow quality to multiply quantity in the utility function, leading to a sub-problem of quality choice for the firm: to minimize the average cost of quality. As in Verhoogen (2008), we assume a Cobb-Douglas production function for quality where firms can differ in their productivities, and let $\theta < 1$ denote the elasticity of quality with respect to the aggregate input used to produce quality. Then we find that quality is a simple log-linear function of firm’s productivity and the aggregate input price, as well as the specific transport costs to the destination market. Specializing to the CES demand system, we solve for the prices charged by firms and find that an exporter’s f.o.b. price is proportional to specific transport costs, as in the Washington apples effect. So up to a constant, log quality is proportional to the log of the exporter’s f.o.b. price divided by the productivity-adjusted input price.

In order to implement this measure of quality, we therefore need accurate information on the inputs used to produce quality as well as the productivity of firms to each export market. Verhoogen (2008) argues that skilled labor is needed to produce high-quality outputs, while De Loeker and Warzynski (2011) further argue that it is important to model all the inputs used by a firm to measure productivity, especially for exporters. The ability to obtain data on these input prices for a broad range of industries (all disaggregate merchandise exports) and countries (all
countries included in the Penn World Table), as is our goal here, is a formidable challenge. To overcome this challenge, we rely on the equilibrium assumption that the marginal exporting firm to each destination market earns zero profits, as in Melitz (2003) and Baldwin and Harrigan (2011). We further assume that the distribution of productivities across firms is Pareto, with a common parameter across countries, and that the fixed cost of selling to each destination market equals the productivity-adjusted input price times a destination-specific factor. Then we can use the zero-cutoff-profit condition to solve for the productivity-adjusted price of inputs, in terms of the exporter’s f.o.b. price and the tariff to each destination market.

In section 3, we aggregate these firm-level results to the product level, in which case the c.i.f. and f.o.b. prices are measured by unit-values. The CES demand system demand depends negatively on the c.i.f. unit value of a product, and should depend positively on exporter’s f.o.b. unit-value, which measures quality up to a factor of proportionality depending on $\theta$ and the elasticity of substitution $\sigma$. That demand system combined with a supply equation governing the specific transport costs between countries enables us to estimate these parameters. In section 4-5, we estimate the demand system using detailed bilateral trade data at the with SITC 4-digit level (about 1,000 products per year) for over 100 countries for 1984-2008. In addition to estimating the key parameter $\theta$, our estimates of the elasticity of substitution $\sigma$ can be compared to those in Broda and Weinstein (2006). Our estimates correct for potential correlation between demand and supply errors due to quality, and differ based on estimation in levels versus first-differences. While Broda and Weinstein (2006) used first-differences following Feenstra (1994), our reliance on the Washington’s apples effect here suggests that levels are more appropriate (since the distance to destination countries is lost when data are first-differenced). In fact, we find that estimates of $\sigma$ are higher when correcting for quality and estimating in levels.
Given the estimates of $\theta$ and $\sigma$, product quality and quality-adjusted prices are readily constructed. Our interest in these is not just academic, but serve a very practical goal: to extend the Penn World Table (PWT) to incorporate the prices of traded goods. As described in Feenstra et al (2009), the prices of internationally traded goods can be used to make a distinction between real GDP on the expenditure-side and real GDP on the output-side: these differ by country’s terms of trade. But that distinction can be made only if the trade unit values are first corrected for quality. That is the goal of this study, and in section 6 we briefly described how the quality-adjusted prices will be incorporated in the next generation of PWT.

2. Optimal Quality Choice

Consumer Problem

Suppose that consumers in country $k$ have available $i=1,\ldots,N^k$ varieties of a differentiated product. These products can come from different source countries (including country $k$ itself). We should really think of each variety as indexed by the triple $(i,j,t)$, where $i$ is the country of origin, $j$ is the firm and $t$ is time. But initially, we will simply use the notation $i$ for product varieties. Firms make the optimal choice of the quality $z_i^k$ to send to country $k$. We will suppose that the demand for the products in country $k$ arises from utility function $U(z_1^k c_1^k, \ldots, z_{N^k}^k c_{N^k}^k)$ where quality $z_i^k$ multiplies the quantity $c_i^k$. Later we will specialize to the CES form:

$$U(z_1^k c_1^k, \ldots, z_{N^k}^k c_{N^k}^k) = \sum_{i=1}^{N^k} \left( \frac{z_i^k c_i^k}{\sigma} \right)^{(\sigma-1)/\sigma}, \quad \sigma > 1.$$  

(1)

We suppose there are both specific and ad valorem trade costs between the countries, which include transportation costs and tariffs. Specific trade costs are given by $T_i^k$, which depend on
the distance to the destination market \( k \). One plus the *ad valorem* trade costs are denoted by \( \tau_i^k \), and for convenience we assume that these are applied to the price *inclusive* of the specific trade costs.\(^3\) Then letting \( p_i^k \) denote the exporters’ f.o.b. price, the tariff-inclusive c.i.f. price is

\[
P_i^k = \tau_i^k (p_i^k + T_i^k).
\]

Thus, consumers in country \( k \) are presented with a set of \( i = 1, \ldots, N^k \) varieties, with characteristics \( z_i^k \) and prices \( P_i^k \), and then choose the optimal quantity of each variety. It will be convenient to work with the *quality-adjusted, tariff-inclusive c.i.f. prices*, which are defined by

\[
\pi_i^k = P_i^k / z_i^k = \tau_i^k (p_i^k + T_i^k) / z_i^k.
\]

The higher is overall product quality \( z_i^k \), *ceteris paribus*, the lower are the quality-adjusted prices \( \pi_i^k \). The consumer maximizes utility subject to the budget constraint 

\[
\sum_{i=1}^{N^k} \tau_i^k (p_i^k + T_i^k) c_i^k \leq Y^k.
\]

The Lagrangian for country \( k \) is,

\[
L = U(z_1^k c_1^k, \ldots, z_{N^k}^k c_{N^k}^k) + \lambda [Y^k - \sum_{i=1}^{N^k} \tau_i^k (p_i^k + T_i^k) c_i^k]
\]

\[
= U(d_1^k, \ldots, d_{N^k}^k) + \lambda (Y^k - \sum_{i=1}^{N^k} \pi_i^k d_i^k),
\]

where the second line of (2) follows by defining \( d_i^k = z_i^k c_i^k \) as the *quality-adjusted demand*, and also using the quality-adjusted prices \( \pi_i^k = \tau_i^k (p_i^k + T_i^k) / z_i^k \). This re-writing of the Lagrangian makes it clear that instead of choosing \( c_i^k \) given c.i.f. prices \( \tau_i^k (p_i^k + T_i^k) \) and quality \( z_i^k \), we can instead think of the representative consumer as choosing \( d_i^k \) given quality-adjusted c.i.f. prices \( \pi_i^k \), \( i = 1, \ldots, N^k \). Let us denote the solution to problem (2) by \( d_i(\pi^k, Y^k) \), \( i = 1, \ldots, N^k \), where \( \pi^k \) is the vector of quality-adjusted prices.

\(^3\) Many countries apply tariffs to the transport-inclusive (c.i.f.) price of a product.
Firms’ Problem

We now add the subscript $j$ for firms, while $i$ denotes their country of origin, so that $(i,j)$ denotes a unique variety. We will denote the range of firms exporting from country $i$ to $k$ by $j = 1, \ldots, N_i^k$. We assume that the input $x_{ij}^k$ needed to produce one unit of a good with product quality $z_{ij}^k$ arises from a Cobb-Douglas function:

$$z_{ij}^k = (x_{ij}^k \varphi_{ij})^\theta,$$

where $0 < \theta < 1$ reflects diminishing returns to quality and $\varphi_{ij}$ denotes the productivity of firm $j$ in country $i$.\(^4\) We think of $x_{ij}^k$ as an aggregate of inputs, including skilled labor as in Verhoogen (2008) but other factors as well, and denote its aggregate price by $w_i$. The marginal cost of producing a good of quality $z_{ij}^k$ is then,

$$g_{ij}(z_{ij}^k, w_i) = w_i x_{ij}^k = w_i (z_{ij}^k)^{1/\theta} / \varphi_{ij}. \quad (4)$$

Firms simultaneously choose f.o.b. prices $p_{ij}^k$ and characteristics $z_{ij}^k$ for each destination market. Then the profits from exporting to country $k$ are:

$$\max_{p_{ij}^k, z_{ij}^k} \left[ p_{ij}^k - g_{ij}(z_{ij}^k, w_i) \right] e_{ij}^k = \max_{p_{ij}^k, z_{ij}^k} \left[ \frac{p_{ij}^k}{z_{ij}^k} - \frac{g_{ij}(z_{ij}^k, w_i)}{z_{ij}^k} \right] d_{ij}^k (\pi^k, y^k) = \max_{\tau_i^k, z_{ij}^k} \left\{ \frac{\tau_i^k}{\tau_i^k} - \frac{[g_{ij}(z_{ij}^k, w_i) + T_i^k]}{z_{ij}^k} \right\} d_{ij}^k (\pi^k, y^k). \quad (5)$$

\(^4\) Verhoogen (2008) models product quality as a function of the quality of several inputs: skilled labor; production labor; equipment; and the entrepreneur.
The first equality in (5) converts from observed to quality-adjusted consumption, while the second line converts to quality-adjusted, tariff-inclusive, c.i.f. prices $\pi^k_{ij} = \tau^k_i (p^k_{ij} + T^k_i) / z^k_{ij}$, along with demands $d^k_{ij}$. The latter transformation relies on our assumption that *prices and characteristics are chosen simultaneously*, as well as our assumption that quality multiplies quantity in the utility function (but (5) does not rely on the CES form in (1)).

It is immediate that to maximize profits in (5), the firms must choose $z^k_{ij}$ to minimize $[g^k_{ij}(z^k_{ij}, w_i) + T^k_{ij}] / z^k_{ij}$, which is interpreted as the *minimizing the average cost per unit of quality inclusive of specific trade costs*. The same optimality condition appears in Rodriguez (1979), who also assumes that quantity multiplies quality in the utility function. Differentiating this objective w.r.t. $z^k_{ij}$, we obtain the first-order condition:

$$\left[ g^k_{ij}(z^k_{ij}, w_i) + T^k_{ij} \right] / z^k_{ij} = \frac{\partial g^k_{ij}(z^k_{ij}, w_i)}{\partial z^k_{ij}}.$$  \hfill (6)

so that the average cost equals the marginal cost when average costs are minimized. The second-order condition for this cost-minimization problem is that $\partial^2 g^k_{ij} / \partial (z^k_{ij})^2 > 0$, so there must be increasing marginal costs of improving quality. An increase in the distance to the destination market raises $T^k_{ij}$, so to satisfy (6) firms will choose a higher quality $z^k_{ij}$, as readily shown from $\partial^2 g^k_{ij} / \partial (z^k_{ij})^2 > 0$. This is the well-known “Washington apples” effect, whereby higher quality goods are sent to more distant markets.

Making use of the Cobb-Douglas production function for quality in (3), and associated cost function in (4), the second-order conditions are satisfied when $0 < \theta < 1$, which we have already assumed. The first-order condition (6) can be simplified as:
\[
\ln z_{ij}^k = \theta \left[ \ln T_i^k - \ln(w_i / \varphi_{ij}) + \ln(\theta / (1 - \theta)) \right].
\] (7)

Conveniently, the Cobb-Douglas production function and specific trade costs give us a log-linear form for the optimal quality choice. We see that more distant markets, with higher transport costs \(T_i^k\), will have higher quality, but that log quality is only a fraction \(\theta < 1\) of the log transport costs. In addition, higher firm productivity \(\varphi_{ij}\) leads to lower effective wages \((w_i / \varphi_{ij})\), and also leads to higher quality. Finally, substituting (7) into the cost function (4), we immediately obtain \(g_{ij}(z_{ij}^k, w_i) = [\theta / (1 - \theta)]T_i^k\). Thus, the marginal costs of production are proportional to the specific trade costs, which we shall use repeatedly.

Now suppose that demand \(d_{ij}^k\) arises from the CES utility function in (1). Solving (3) for the optimal choice of the quality-adjusted price \(q_{ij}^k\), we obtain the familiar markup:

\[
(p_{ij}^k + T_i^k) = [g_{ij}(z_{ij}^k, w_i) + T_i^k] \left( \frac{\sigma}{\sigma - 1} \right).
\]

This equation shows that firms not only markup over marginal costs \(g_{ij}\) in the usual manner, they also markup over specific trade costs. Then using the relation \(g_{ij}(z_{ij}^k, w_i) = [\theta / (1 - \theta)]T_i^k\), we readily solve for the f.o.b. and tariff-inclusive c.i.f. prices as:

\[
\ln p_{ij}^k = \ln T_i^k + \ln \left[ \left( \frac{1}{1 - \theta} \right) \left( \frac{\sigma}{\sigma - 1} \right) - 1 \right] = \ln \bar{p}_i^k,
\] \hspace{1cm} (8a)

\[
\ln P_{ij}^k = \ln \tau_i^k + \ln T_i^k + \ln \left[ \left( \frac{1}{1 - \theta} \right) \left( \frac{\sigma}{\sigma - 1} \right) \right] = \ln \bar{P}_i^k.
\] \hspace{1cm} (8b)

Thus, both the f.o.b. and c.i.f. prices vary across destination markets \(k\) in direct proportion to the specific transport costs to each market, and are independent of the productivity of the firm \(j\), as indicated by the notation \(\ln \bar{p}_i^k\) and \(\ln \bar{P}_i^k\). This result is obtained because more efficient firms
sell higher quality goods, leading to constant prices to each destination market.

Combining (7) and (8) we obtain:

$$\ln z^k_{ij} = \theta \left[ \ln \bar{p}^k_i - \ln (w_i / \varphi_{ij}) \right] + \kappa_1,$$

where \( \kappa_1 \) is a parameter depending on \( \theta \) and \( \sigma \). Thus, quality \( z^k_{ij} \) depends on the ratio of the f.o.b. price \( \bar{p}^k_i \) to the productivity-adjusted input price \( (w_i / \varphi_{ij}) \) of the exporter. It follows that the quality-adjusted price \( \pi^k_{ij} = \bar{p}^k_i / z^k_{ij} \) is:

$$\ln \pi^k_{ij} = \ln \bar{p}^k_i - \theta \left[ \ln \bar{p}^k_i - \ln (w_i / \varphi_{ij}) \right] - \kappa_1. \quad (10)$$

Since from (8) the c.i.f. and f.o.b. prices do not differ across firms selling to each destination market, then the quality-adjusted price is decreasing in the productivity \( \varphi_{ij} \) of the exporter, as in the original Melitz (2003) model.

**Zero-Cutoff-Profit Condition**

As discussed in section 1, it would be a formidable challenge to assemble the data on input prices and firms’ productivities needed to measure quality in (9) across many goods and countries, as is our goal here. Accordingly, we rely instead on the zero-cutoff-profit (ZCP) condition of Melitz (2003) to solve for the productivity-adjusted input price of the marginal exporter to each destination market.

Making use of (7) – (10), the quality-adjusted price net of the tariff can be expressed as,

$$\frac{\pi^k_{ij}}{\tau^k_i} = \left( \frac{\bar{p}^k_i}{\tau^k_i} \right)^{1-\theta} \left( \frac{w_i}{\varphi_{ij}} \right)^\theta \kappa_2,$$
where $\kappa_2$ is a parameter depending on $\theta$ and $\sigma$. It is similarly shown that the quality-adjusted cost of producing each unit, inclusive of the specific transport costs is,

$$
\frac{g_{ij}(z^k_{ij}, w_i) + T^k_i}{z^k_{ij}} = \left( \frac{p^k_i}{\phi_{ij}} \right)^{1-\theta} \left[ w_i \left( \frac{\sigma - 1}{\sigma} \right) \right]^{\kappa_2}.
$$

Notice that to obtain profits in (5), we take the difference between these two terms and multiply by demand $d_{ij}^k(\pi^k, y^k)$. For the CES demand system in (1), the ratio of demand for two firms is

$$
d_{ij}^k / d_{lm}^k = (\pi_{ij}^k / \pi_{lm}^k)^{-\sigma}.
$$

Let us suppose that a firm with the productivity-adjusted input price $(w_i / \varphi_{ij})$ has the fixed cost $f^k(w_i / \varphi_{ij})$ to sell to destination market $k$, where $f^k$ is common to all firms exporting to $k$. Then by setting profits equal to fixed costs for the cutoff firms $\hat{\phi}_{ij}^k$ and $\hat{\phi}_{lm}^k$, we can readily solve for the ratio of productivity-adjusted input prices as:

$$
\left( \frac{w_i / \hat{\phi}_{ij}^k}{w_l / \hat{\phi}_{lm}^k} \right) = \left( \frac{p^k_i}{p^k_l} \right)^{\frac{\sigma}{1 + (\sigma - 1)\theta}} \left( \frac{T^k_i}{T^k_l} \right)^{-\sigma}.
$$

(11)

Notice that the exponent of the f.o.b. price ratio in this expression is less than zero, since $\sigma > 1 + (\sigma - 1)\theta$, so that higher-prices are associated with lower productivity-adjusted input prices. That results in higher quality, as can be seen by substituting (11) into (9) to obtain the ratio of qualities for the cutoff exporters:

$$
\frac{z_{ij}^k}{z_{lm}^k} = \left( \frac{p^k_i \tau_{ij}^k}{p^k_l \tau_{ij}^k} \right)^{\frac{\sigma \theta}{1 + (\sigma - 1)\theta}}.
$$

(12)

We see that the cutoff levels of quality depend on both the f.o.b. price and the ad valorem tariff, with an exponent that is less than unity. Normally, the ad valorem tariff does not affect the choice of quality, as can be seem from the first-order condition (6) which is independent of the
tariff. But this tariff still affects the cutoff level of quality through a selection effect, as emphasized by Baldwin and Harrigan (2011): only firms with high enough productivity will be exporters, and their corresponding choice of quality is influenced by their productivity.\footnote{That is, while Baldwin and Harrigan (2011, section III) do not have firms endogenously choosing quality, they still obtain a “Washington apples” effect because only the highest productivity firms – which also have high quality by assumption – ship to the furthest markets. Such a selection effect on quality also holds in our model, in additional to the endogenous choice of quality in (6). Harrigan and Shlychkov (2010) argue that for U.S. exporters the selection effect is the only factor leading to quality differences across destination markets; but different results are obtained for Portuguese exporters by Bustos and Silva (2010).}

Exporters not at the margin with higher productivity will have corresponding higher quality. In the next section we integrate over the set of exporters from each source to each destination country, to obtain average quality and quality-adjusted prices.

3. Aggregation and Demand

In the equations above we explicitly distinguish firms $j$ in each country $i$, but in our data we will not have firm-level information for every country. Accordingly, we need to aggregate to the product level, and following Melitz (2003), we form the CES averages of relevant variables. To achieve this, let us add the assumption that firms productivity is Pareto distribution with a continuum of firms in each country. Denoted this distribution function by $G(\varphi) = 1 - \varphi^{-\gamma}$ with density $g(\varphi) = -\varphi^{-(\gamma+1)}$, and CES-average of productivity-adjusted input price for all firms in country $i$ exporting to country $k$ is:

$$\hat{w}_{ik} = \frac{1}{\phi_i} \left[ \frac{1}{\phi_j} \left( \frac{w_i}{\varphi} \right)^{1-\sigma} g(\varphi) \right]^{1-\sigma} \left[ \frac{1}{1 - G(\varphi)} \right]^{1-\sigma} \frac{w_i}{\phi_j},$$

as obtained by evaluating the integral and assuming that $\gamma > (\sigma - 1)\theta$. This expression shows a convenient property of the Pareto distribution, whereby the integral from a cutoff to infinity of a
power-function of productivity is proportional to that function evaluated at the cutoff. We assume that the Pareto parameter is common across countries, so the factor of proportionality cancels when taking ratios. We therefore find that the expression in (11) holds equally well for the ratio of productivity-adjusted input prices for the average exporters from countries \( i \) and \( l \) selling to country \( k \). Likewise, (12) holds for the ratio of average quality \( (z_i^k / z_l^k) \) for countries \( i \) and \( l \) selling to country \( k \). It follows that the ratio of average quality-adjusted prices is:

\[
\frac{p_i^k}{p_l^k} = \left( \frac{\pi_i^k}{\pi_l^k} \right)^{\sigma \theta \tau} \quad \text{(13)}
\]

Because we have aggregated over firms, for convenience we now let the subscript \( j \) denote another country, and also add a time subscript \( t \). Then for the CES utility function in (1), the share of expenditure in country \( k \) spent on varieties from country \( i \), denoted by \( s_{it}^k \), relative to the share spent on varieties from country \( j \), is:

\[
\ln s_{it}^k - \ln s_{jt}^k = - (\sigma - 1) \left( \ln \pi_{it}^k - \ln \pi_{jt}^k \right) + \ln N_{it}^k - \ln N_{jt}^k, \quad \text{(14)}
\]

where \( N_{it}^k \) and \( N_{jt}^k \) are the number of firms – or product varieties – exported from country \( i \) and \( j \) to country \( k \). The intuition for (14) is that if there are more firms/product varieties selling from country \( i \) to \( k \) then the share of demand \( s_{it}^k \) will by higher. The presence of these product variety terms plagues all attempts to measure quality, because either greater variety or higher quality (leading to lower quality-adjusted prices) will raise demand. This problem is dealt with in different ways by Hallak and Schott (2011), Hummels and Klenow (2005), and Khandelwal (2010): the latter author, for example, uses exporting country population to measure \( N_{it}^k \). We
will suppose instead that variety depends on country fixed effects, distance, and tariffs, in a gravity-type equation:

$$\ln N_{it}^k = \alpha_i + \alpha^k + \beta_1 \text{dist}_{it}^k + \beta_2 \ln \tau_{it}^k + \epsilon_i^k. \quad (15)$$

Substituting (15) and the average quality-adjusted prices from (12) into the demand equation (14), we obtain:

$$\ln s_{it}^k - \ln s_{jt}^k = -(\sigma - 1) \left\{ \left[ \ln \bar{P}_{jt}^k - \ln \bar{p}_{it}^k \right] - \frac{\sigma \theta}{1 + (\sigma - 1) \theta} \left[ \ln (\bar{p}_{it}^k \tau_{it}^k) - \ln (\bar{p}_{jt}^k \tau_{jt}^k) \right] \right\}$$

$$+ \alpha_i - \alpha_j + \beta_1 (\text{dist}_{it}^k - \text{dist}_{jt}^k) + \beta_2 (\ln \tau_{it}^k - \tau_{jt}^k) + \epsilon_i^k - \epsilon_j^k. \quad (16)$$

In this demand equation, the tariff-inclusive c.i.f. prices $\bar{P}_{it}^k$ enter with a negative coefficient, but the tariff-inclusive f.o.b. prices $(\bar{p}_{it}^k \tau_{it}^k)$ enter with a positive coefficient. This sign pattern arises because the f.o.b. prices are capturing quality. The empirical challenge will be to obtain the expected signs can be obtained on these two prices, while also controlling for the endogeneity of shares and prices.

4. Estimation

Our goal is to estimate equation (16) to obtain estimates of $\theta$ and $\sigma$, while recognizing that the shares and prices appearing there are endogenous. To control for this endogeneity we will modify the GMM methodology introduced by Feenstra (1994). That methodology exploits the moment condition that the error in demand and supply are uncorrelated. That assumption could be violated when quality is present, however, since a change in quality could act as shift to both supply and demand. While that criticism can be made of Feenstra (1994) and Broda and Weinstein (2003), it does not apply here because we have explicitly modeled quality choice. To complete our model, we need to develop the supply side in more detail.
The f.o.b. prices are shown in (8a), depending on the specific transport costs and a markup. We shall assume that the specific transport costs depend on distance and a measure of the aggregate quantity \( \tilde{d}_{ik}^k = \frac{y_{ik}^k}{\pi_{ik}^k} \) exported from country \( i \) to \( k \):

\[
\ln T_{ik}^k = \eta_d + \eta_d \text{dist}_i^k + \omega \ln \tilde{d}_{ik}^k + \epsilon_{ik}^k.
\]

We are including the quantity \( \tilde{d}_{ik}^k \) exported to reflect possible congestion in shipping, but also so that our model here nests that used in Feenstra (1994), who likewise assumed an upward sloping supply curve. We also suppose that transport costs depend on a global time trend \( \eta_t \), which can reflect productivity, and a random error \( \epsilon_{ik}^k \) that we shall treat as independent of \( \epsilon_{ij}^k \).

Combining (17) with (8) and (13), we solve for an inverse supply curve as:

\[
\ln \left( \frac{x_{it}^k}{x_{jt}^k} \right) = \rho (\ln \tau_{it}^k - \ln \tau_{jt}^k) + \eta \rho (\text{dist}_i^k - \text{dist}_j^k) + \omega \rho (\ln \tilde{d}_{it}^k - \ln \tilde{d}_{jt}^k) + \rho (\delta_{et}^k - \delta_{ft}^k),
\]

where \( \rho \equiv (1 - \theta) / [1 + (\sigma - 1)\theta] \). Equations (16) and (18) are the same as the system in Feenstra (1994), except for three features: (i) the price is the quality-adjusted price; (ii) the presence of tariffs and distance the right-hand side of both equations; (iii) we do not express the system in first-differences over time, because we want to retain distance as a variable that is important for the choice of quality. As in Feenstra (1994), we simplify (18) by using the share to replace the quantity \( \tilde{d}_{it}^k = \frac{y_{ik}^k}{\pi_{ik}^k} \). Expressing both equations with their errors and exogenous variables on the left, we can obtain (see the Appendix):

\[
(\epsilon_{it}^k - \epsilon_{jt}^k) + (\alpha_i - \alpha_j) + \beta_1 (\text{dist}_i^k - \text{dist}_j^k) + \beta_2 (\ln \tilde{d}_{it}^k - \ln \tilde{d}_{jt}^k)
= \left( \ln s_{it}^k - \ln s_{jt}^k \right) + (\sigma - 1) \left( \ln \pi_{it}^k - \ln \pi_{jt}^k \right)
\]

Note that the supply curve for transportation services could instead by downward sloping, with \( \omega < 0 \).
\[
\rho(\delta_{it}^k - \delta_{jt}^k) + \eta \rho(\text{dist}_{it}^k - \text{dist}_{jt}^k) + \rho(\ln \tau_{it}^k - \ln \tau_{jt}^k) \\
= (1 + \omega \rho)\left(\ln \pi_{it}^k - \ln \pi_{jt}^k\right) - \omega \rho(\ln s_{it}^k - \ln s_{jt}^k).
\] (20)

Multiplying these two equations and dividing by \((1 + \omega \rho)(\sigma - 1)\) gives a lengthy equation, reported in the Appendix, which has an error depending on the product \((\varepsilon_{it}^k, \delta_{it}^k)\) and variables that are the second moments and cross-moments of the data. This is the analogue to the demand and supply system in Feenstra (1994), extended here to endogenous quality choice. Feenstra (1994) assumed that the supply shocks are uncorrelated with the demand shocks. That assumption is unlikely to hold with unobserved quality, however, since a change in quality could shift both supply and demand. But in this paper, the errors \(\varepsilon_{it}^k\) and \(\delta_{it}^k\) are the residuals in demand (16) and supply (18) after taking into account quality. The assumption that \(\varepsilon_{it}^k\) and \(\delta_{it}^k\) are uncorrelated therefore seems much more acceptable.

6. Data

The primary dataset used is the United Nations’ Comtrade database. We obtain bilateral f.o.b. prices of traded goods by calculating the unit value of each bilateral transaction at the four-digit SITC industry level, as reported by the exporting country. By focusing on the exporters’ reports we ensure that these values are calculated prior to the inclusion of any costs of shipping the good. The bilateral c.i.f. prices are then calculated similarly using importers’ reports of the value of the good. Since this value includes the costs of shipping, we need only to add the value of any tariff on the good to produce a tariff-inclusive c.i.f. price. To do this we obtain tariff values associated with Most Favored Nation status or any preferential status from TRAINS, which we have expanded upon using tariff schedules from the *International Customs Journal*. 


and the texts of preferential trade agreements obtained from the World Trade Organization's website and other online sources.

7. Estimation Results

Table 1 and Figures 1 and 2 summarize our regression results. The median sigma estimate is 9.4, and the median standard error of our sigma estimates is 0.12. We do not consider the mean sigma to be a useful statistic, driven as it is by very high estimates for highly substitutable goods. We instead report the mean estimated markup, at 12 percent (0.12). 1184 of the 1187 sigma estimates have admissible values (>1). Figure 1 summarizes the distribution of these estimates, where for the purposes of this figure only, estimates greater than 26 have been censored at 26. We do not adopt the grid-search algorithm used in Broda and Weinstein (2006) to replace inadmissible values. Instead, we replace inadmissible estimates with neighboring estimates, such as the median admissible sigma for the corresponding SITC 3-digit level. Occasionally, we have to employ the median SITC 2-digit estimate.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>σ</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>9.4</td>
<td>0.14</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.12</td>
<td>0.002</td>
</tr>
<tr>
<td>Mean 1/(σ-1)</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>Median Varieties Per Category</td>
<td>1464</td>
<td>1464</td>
</tr>
<tr>
<td>Categories</td>
<td>1187</td>
<td>1187</td>
</tr>
</tbody>
</table>
Figure 1: Estimates of $\sigma$

Figure 2: Estimates of $\theta$
Our median theta estimate is 0.14, with a median standard error of 0.002. Only 31 of our estimates lie outside the interval [0,1] and are therefore inadmissible, and for these we again substitute neighboring estimates. Figure 2 shows the distribution of our theta estimates.

Our sigma estimates are noticeably higher than those obtained by Broda and Weinstein (2006). Based on Table IV of their paper, we would expect to obtain a median elasticity at the SITC 4-digit level between 2.5 and 2.8. We instead get 9.4. While 9.4 may at first seem high, it is closer to estimates obtained by researchers that use detailed trade data to identify "long-run" elasticities - see for example Romalis (2007). We reconcile the differences with Broda and Weinstein in Table 2. It is not the different data source that is responsible for this difference - when we estimate a model without quality on US imports only (as in Broda-Weinstein) we obtain a median elasticity of 2.7. We then extend our analysis to all bilateral trade, further raising the median sigma to 4.7. Since we have trade reports from both the exporting and the importing country we can drop "unreliable" observations which are most subject to measurement or reporting error and are likely to attenuate estimates of sigma. Simply dropping the 5 percent of observations with the largest log-difference between reported unit values in the exporter's report and the importer's report raises our typical sigma to 5.4. Estimation in levels rather than in differences raises our median estimate to 8.0. This could be due to two factors: (i) the attenuation bias from measurement error in the data is likely to be magnified by first-differencing; and (ii) sigma may be higher in the long-run than in the short-run, which will be partly captured by our levels estimation. Finally, explicitly modeling quality raises our median estimate to 9.4. Without this last step, quality improvements are falsely interpreted as price increases, biasing downwards estimates of sigma.

7 Ideally, to estimate both short-run and long-run elasticities we would model the dynamics of demand responses to price changes.
Table 2: Reconciling Our Estimates to Broda and Weinstein (2006)

<table>
<thead>
<tr>
<th>Estimation Method</th>
<th>Median Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA Imports Only</td>
<td>2.7</td>
</tr>
<tr>
<td>+ All Bilateral Trade</td>
<td>4.7</td>
</tr>
<tr>
<td>+ Dropping Unreliable Observations</td>
<td>5.4</td>
</tr>
<tr>
<td>+ Estimating in Levels</td>
<td>8.0</td>
</tr>
<tr>
<td>+ Modeling Quality</td>
<td>9.4</td>
</tr>
</tbody>
</table>

{add material on methodology used to construct price indexes; perform aggregation for other years}

Figure 3 is a first attempt to check whether our quality estimates for each exporting country conform to expectations. For each 4-digit SITC product, we construct a relative price index for exports and then a similar relative quality measure based on our quality estimates. We report these indexes for 2005 for all countries then in the PWT. The data broadly conforms with our priors. Developed countries tend to export more expensive goods (top panel), and these goods are estimated to be of higher than average quality (second panel). The quality adjusted-price (price less quality), about which we have less strong priors, tends to be only slightly higher for developed countries (bottom panel).
We illustrate a similar exercise for import prices in Figure 4. Developed countries import more expensive items (top panel) that are of higher quality (second panel). Quality-adjusted import prices increase moderately with the importing country's GDP per capita.

Figure 5 shows our preliminary terms of trade estimates for 2005. Terms of trade estimates using unadjusted export and import prices fluctuate substantially across countries, and lie between 0.57 and 1.32. Terms of trade estimated constructed from quality-adjusted prices move in a much narrower band, between 0.77 and 1.12. Despite the narrowness of this band, these quality-adjusted terms of trade measures are sufficiently different from 1 to produce meaningful differences between output-based and expenditure-based real GDP estimates for many countries in the PWT.
Figure 3: Exports - Raw Prices, Quality, and Quality Adjusted Prices in 2005
Figure 4: Relative Import Unit Values, Quality, and Quality Adjusted Prices in 2005
8. Conclusion

Our goal to adjust observed trade unit values for quality to estimate trade prices. Trade price estimates will be used to construct an output-based measure of real GDP in the Penn World Table. We achieve this result by explicitly modeling quality-choice by exporting firms. Our key parameter estimate of the elasticity of quality with respect to the quantity of inputs almost always lies between 0 and 1, as required by our model. Our estimates of the elasticity of substitution
between different varieties of the same SITC 4-digit products are substantially higher than in Broda-Weinstein, and the differences are large enough to greatly affect any trade or welfare calculations that employ these estimates. We reconcile our estimates with BW and account for why we get higher estimates. Finally, we use our estimates to construct preliminary quality and trade price estimates.
Appendix:

The demand equation (16) can be re-written as (19). Substituting \( \overline{d}^k_{it} \equiv Y^k s^k_{it} / \pi^k_{it} \) into the supply equation (18), we obtain:

\[
\ln \pi^k_{it} - \ln \pi^k_{jt} = \omega \rho \left( \ln \frac{Y^k s^k_{it}}{\pi^k_{it}} + \ln \frac{Y^k s^k_{jt}}{\pi^k_{jt}} \right) + \rho (\ln \tau^k_{it} - \ln \tau^k_{jt}) + \eta \rho (dist^k_i - dist^k_j) + \rho (\delta^k_i - \delta^k_j) \]

\[
= \omega \rho \left( \ln \pi^k_{it} - \ln \pi^k_{jt} \right) + \rho (\ln \tau^k_{it} - \ln \tau^k_{jt}) + \eta \rho (dist^k_i - dist^k_j) + \rho (\delta^k_i - \delta^k_j) \]

\[
= -\omega \rho \sigma \left( \ln \pi^k_{it} - \ln \pi^k_{jt} \right) + \omega \rho (\alpha_i - \alpha_j) + \rho (\eta \beta_1 \omega (dist^k_i - dist^k_j) + \rho (1 + \beta_2 \omega) (\ln \tau^k_{it} - \ln \tau^k_{jt}) + \rho (\delta^k_i - \delta^k_j), \]

where the last equality substitutes for the shares from (16). It follows that:

\[
\ln \pi^k_{it} - \ln \pi^k_{jt} = (\alpha_i' - \alpha_j') + \beta_i' (dist^k_i - dist^k_j) + \beta_2' (\ln \tau^k_{it} - \ln \tau^k_{jt}) + \frac{\omega \rho (\epsilon^k_{it} - \epsilon^k_{jt})}{(1 + \omega \rho \sigma)} + \frac{\rho (\delta^k_i - \delta^k_j)}{(1 + \omega \rho \sigma)}, \]

where \( \alpha_i' = \frac{\omega \rho}{(1 + \omega \rho \sigma)} \alpha_i, \) \( \beta_i' = \frac{\rho (\eta + \omega \beta_1)}{(1 + \omega \rho \sigma)}, \) and \( \beta_2' = \frac{\rho (1 + \omega \beta_2)}{(1 + \omega \rho \sigma)}. \) This is a reduced-form supply curve. Substituting for \( (\epsilon^k_{it} - \epsilon^k_{jt}) \) from (16) leads to:

\[
\ln \pi^k_{it} - \ln \pi^k_{jt} = (\alpha_i' - \alpha_j') + \beta_i' (dist^k_i - dist^k_j) + \beta_2' (\ln \tau^k_{it} - \ln \tau^k_{jt}) + \frac{\rho (\delta^k_i - \delta^k_j)}{(1 + \omega \rho \sigma)} + \frac{\omega \rho}{(1 + \omega \rho \sigma)} \left[ \ln s^k_{it} - \ln s^k_{jt} + (\sigma - 1) \left( \ln \pi^k_{it} - \ln \pi^k_{jt} \right) - (\alpha_i - \alpha_j) - \beta_1 (dist^k_i - dist^k_j) - \beta_2 (\ln \tau^k_{it} - \ln \tau^k_{jt}) \right]. \]

Since \( 1 - \frac{\omega \rho (\sigma - 1)}{(1 + \omega \rho \sigma)} = \frac{1 + \omega \rho}{(1 + \omega \rho \sigma)}, \) this equation can be simplified as (20), shown in the text.
\[
\left( \ln \pi^k_{it} - \ln \pi^k_{jt} \right)^2 \\
= \frac{\varphi \rho}{(1 + \varphi \rho)(\sigma - 1)} \left( \ln s^k_{it} - \ln s^k_{jt} \right)^2 + \left( \frac{\varphi \rho - \frac{1}{\sigma - 1}}{1 + \varphi \rho} \right) \left( \ln \pi^k_{it} - \ln \pi^k_{jt} \right) \\
+ (\alpha_i^* - \alpha_j^*) \eta (\text{dist}^k_{it} - \text{dist}^k_{jt}) + (\alpha_i^* - \alpha_j^*) (\ln \tau^k_{it} - \ln \tau^k_{jt}) + \beta_i^* \eta (\text{dist}^k_{it} - \text{dist}^k_{jt})^2 \\
+ \beta_2^* (\ln \tau^k_{it} - \tau^k_{jt})^2 + (\beta_1^* + \beta_2^* \eta) (\text{dist}^k_{it} - \text{dist}^k_{jt}) (\ln \tau^k_{it} - \ln \tau^k_{jt}) + u^k_{it},
\]

(A1)

where \( \alpha_i^* = \frac{\alpha_i \rho}{(1 + \varphi \rho)(\sigma - 1)} \), \( \beta_1^* = \frac{\beta_1 \rho}{(1 + \varphi \rho)(\sigma - 1)} \), \( \beta_2^* = \frac{\beta_2 \rho}{(1 + \varphi \rho)(\sigma - 1)} \) and the error term is:

\[
u^k_{it} = \frac{\rho (\delta^k_{it} - \delta^k_{jt})}{(1 + \varphi \rho)(\sigma - 1)} \\
\times [(\epsilon^k_{it} - \epsilon^k_{jt}) + (\alpha_i - \alpha_j) + \beta_1 (\text{dist}^k_{it} - \text{dist}^k_{jt}) + \beta_2 (\ln \tau^k_{it} - \tau^k_{jt})] \\
+ \frac{(\epsilon^k_{it} - \epsilon^k_{jt})}{(1 + \varphi \rho)(\sigma - 1)} \left[ \eta \rho (\text{dist}^k_{it} - \text{dist}^k_{jt}) + \rho (\ln \tau^k_{it} - \ln \tau^k_{jt}) \right].
\]

(A2)

Notice that the first three terms in (A1) are similar to those found in Feenstra (1994), except now using the quality-adjusted price. The remaining terms in (A1) are interactions between country fixed effects, distance and tariffs, which enter as controls. These variables also enter the error term in (A2), but because we treat them as exogenous, we can assume that they are uncorrelated with the demand and supply shocks. We further assume that the supply shocks are uncorrelated with the demand shocks, so that \( \mathbb{E} u^k_{it} = 0 \) for each source country \( i \) and destination \( k \). This is the moment condition we use to estimate (A1).

Next, we substitute the quality-adjusted prices from (13) into (A1) to obtain:
\[
\left( \ln \overline{P}^k_{it} - \ln \overline{P}^k_{jt} \right)^2
\]

\[
= 2 \left[ \frac{\sigma \theta}{\sigma \theta + (1 - \theta)} \left( \ln \overline{P}^k_{it} - \ln \overline{P}^k_{jt} \right) \left( \ln \overline{P}^k_{it} - \ln \overline{P}^k_{jt} \right) \right] - \left[ \frac{\sigma \theta}{\sigma \theta + (1 - \theta)} \right]^2 \left( \ln \overline{P}^k_{it} - \ln \overline{P}^k_{jt} \right)^2
\]
\[
+ \frac{\omega \rho}{(1 + \omega \rho)(\sigma - 1)} \left( \ln s^k_{it} - \ln s^k_{jt} \right)^2 + \left( \frac{\omega \rho}{1 + \omega \rho} - \frac{1}{\sigma - 1} \right) \left( \ln s^k_{it} - \ln s^k_{jt} \right) \left( \ln \overline{P}^k_{it} - \ln \overline{P}^k_{jt} \right)
\]
\[
- \left[ \frac{\sigma \theta}{\sigma \theta + (1 - \theta)} \right] \left( \frac{\omega \rho}{1 + \omega \rho} - \frac{1}{\sigma - 1} \right) \left( \ln s^k_{it} - \ln s^k_{jt} \right) \left( \ln \overline{P}^k_{it} - \ln \overline{P}^k_{jt} \right)
\]
\[
+ (\alpha_i^* - \alpha_j^*) \eta (\text{dist}^k_i - \text{dist}^k_j) + (\alpha_i^* - \alpha_j^*) (\ln \tau^k_i - \ln \tau^k_j) + \beta_1^* \eta (\text{dist}^k - \text{dist}^k_j)^2
\]
\[
+ \beta_2^* (\ln \tau^k_i - \ln \tau^k_j)^2 + (\beta_3^* + \beta_2^* \eta) (\text{dist}^k_i - \text{dist}^k_j) (\ln \tau^k_i - \ln \tau^k_j) + u^k.
\]  

(A3)

For estimation, we average the variables in (A3) across source countries \( i \) and destination countries \( k \). This eliminates the time subscript in (A3), and gives a cross-country regression that can be estimated with nonlinear least squares (NLS). A final challenge is to incorporate the country fixed effects \( (\alpha_i^* - \alpha_j^*) \) interacted with distance and tariffs as appear near the end of (A3). The list of countries varies by product, so it is difficult to incorporate these interactions directly into the NLS estimation. Instead, we first regress all other variables in (A3) on those interaction terms, and then estimate (A3) using the residuals obtained from these preliminary regressions.
References


Bekkers, Eddy, Joseph Francois, and Miriam Manchin, 2010, “Import Prices, Income, and Income Inequality,” University of Linz and University College London.


Simonovska, Ina, 2011, “Income Differences and Prices of Tradable,” University of California, Davis.