Inequality, Leverage and Crises*

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Abstract

The paper studies how high leverage and crises can arise as a result of distributional conflict. Empirically, the periods 1920-1929 and 1983-2008 both exhibited a large increase in the income share of the rich, a large increase in leverage for the remainder, and an eventual financial and real crisis. The paper presents a theoretical model where these features arise endogenously as a result of a shift in bargaining power over incomes. A financial crisis can reduce leverage if it is very large and not accompanied by a real contraction. But restoration of the lower income group’s bargaining power is more effective.

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1. Introduction

Over the last century, the United States has experienced two major crises: 1929 and 2008. Both were preceded by a sharp increase in income and wealth inequality, and by a similarly sharp increase in debt leverage among lower and middle income households. In this paper, we first document these facts, and then show how they can arise endogenously in a theoretical model, as a result of a change in bargaining powers over income of different household groups. As the gainers recycle part of what they gained at the expense of the losers, in the form of loans back to the losers, this does allow the latter to maintain their level of consumption despite the loss in income, but it also endogenously generates financial fragility which eventually leads to a financial crisis. Our theoretical framework is a DSGE model which features income heterogeneity, and shocks to bargaining powers over income, across two groups of agents, capital owners and workers. To our knowledge, our framework is the first to provide an internally consistent mechanism linking the empirically observed rise in income inequality between high income households and poor to middle income households, the increase in household debt leverage among the latter group, and the risk of a financial crisis.

Our core results can be summarized as follows. In an economy populated by two income groups - the top 5% and the bottom 95% - a slow-reversing shock to the distribution of incomes in favor of the top group generates a gradual increase of the debt to income ratio of the bottom group, which follows the pattern and magnitudes observed in the data. In our closed economy set-up, the increase in leverage of the bottom 95% is made possible by the re-lending of the increased disposable incomes of the top 5% to the bottom 95%. Increased saving at the top and increased borrowing at the bottom results in consumption inequality increasing significantly less than income inequality. Saving and borrowing patterns of both groups create an increased need for financial services and intermediation. As a consequence the size of the financial sector, as measured by the ratio of banks’ liabilities to GDP, increases. The rise of poor and middle income household debt leverage generates financial fragility and a higher probability of financial crises. In our model, crises are characterized by large-scale household debt defaults and an abrupt output contraction as in the 2007-2008 US financial crisis. Because crises are costly, redistribution policies that prevent excess leverage and reduce crisis-risk ex-ante can be more desirable from a macroeconomic stabilization point of view that ex-post policies such as bailouts or debt restructurings.
This paper integrates two strands of literature that have largely been evolving separately: the literature on income and wealth distribution and the literature on financial fragility and macroeconomic volatility. The first literature is mostly focused on accurately describing long run changes in the distribution of income and wealth (Piketty and Saez (2003), Piketty (2010)). One of its main findings is that the most significant changes in the income distribution concern the evolution of top income shares. This feature is taken on board in our model, where income heterogeneity is introduced by considering two groups representing the top stratum and the remainder of the income distribution. A companion literature in labor economics seeks to uncover the fundamental factors shaping the change in the distribution of earnings in the United States over the last thirty years. It considers the respective roles played by changes in the college premium, changes in the production structure of the U.S. economy, jobs offshoring, or shifts in compensation policies or in the bargaining power of workers and unions. Our paper focuses only on the macroeconomic implications of increased income inequality. Therefore, rather than taking a stance on the microeconomic reasons for that increase, it represents more fundamental shocks by way of a shock to the relative bargaining powers of the two income groups.

The literature on financial fragility has so far ignored the role of income heterogeneity in the exposure to crisis risk. In the canonical Diamond and Dybvig (1983) crisis model, the heterogeneity that matters is that between patient and inpatient consumers, which is not directly observable by financial intermediaries. Differences between impatient and patient consumers also feature prominently in financial accelerator models applied to household debt and housing cycles (Iacoviello (2005, 2008)). In this paper we argue that, because increases in household debt leverage, which increase financial fragility, are strongly heterogenous across income groups, heterogeneity in incomes is a key additional feature that should be introduced into models of household debt leverage and financial crises. In 1983, the top 5% exhibited a debt to income ratio of 80% and the bottom 95% a ratio of 60%. Twenty five years later, the situation is dramatically reversed with a ratio of 65% for the top 5% and of 140% for the bottom 95%.

While not formally modeled there, the link between income inequality, leverage and crises has been recently discussed in opinion editorials by Paul Krugman, and in books by Rajan (2010) and Reich (2010). Both authors suggest that increases in borrowing have been a

\footnote{In the Morris and Shin (1998) set-up, only a vanishingly small heterogeneity in private signals regarding the stability of the financial system is necessary to relate the incidence of crises to economic fundamentals.}
way for the poor and the middle-class to maintain or increase their level of consumption at
times when their real earnings were stalling. We model this feature by introducing a slow
reversing shock to relative bargaining powers over incomes. It has also been suggested that
the increase in wealth of the richest households has played a role in increasing the demand
for investment assets. In our model, the financial sector intermediates funds between the
increasingly richer top fraction of the population and the increasingly more leveraged bottom
fraction of the population. As the flow of funds between the two groups increases, so does
the size of the financial sector as measured by total assets or total liabilities over GDP. This
fact is consistent with recent findings by Philippon (2008). The size of the demand by the
top 5% for bank deposits, in other words for assets backed by household debt, is quantified
by directly introducing wealth into their preferences, reflecting a "capitalist spirit" motive
stressed by a number of authors starting with Carroll (2000). This feature is consistent
with a vast literature that indicates that such a wealth accumulation motive is necessary to
rationalize the saving behavior of the richest households (Dynan, Skinner and Zeldes (2004),
Piketty (2010), Reiter (2004)).

A recent literature has attempted to relate the rise in income inequality to the increase
in household debt (Krueger and Perri (2006), Iacoviello (2008)). There is an important
difference between our approach and that followed by these authors. In their approach
an increase in the variance of idiosyncratic income shocks generates a higher demand for
insurance in credit markets, thereby increasing household debt. By contrast our paper focuses
on the rise of inequality between two income groups, that is on the divergence in the level of
earnings between the two groups, not on an increase in the variance of the idiosyncratic shocks
experienced equally within each group. There is a lively academic debate concerning the
relative roles of within and between group factors in shaping inequality. But one observation
is incontestable: Over the past 30 years there has been a very strong divergence in the trends
of incomes and debt to income ratios between high income households and everyone else.
Our model focuses squarely on that one fact and its implications, and it provides a simple
explanation that has the advantage of realism and simplicity.

Our paper uncovers a new channel by which increases in income inequality affect macro-
economic stability. Some political economy literature has argued that income inequality
can generate political instability, which in turn can destabilize the economy (Benhabib and
Przeworski (2006)). By contrast our channel is much more direct: income inequality gen-
erates macroeconomic volatility through the endogenous response of borrowing and saving
decisions of different groups of agents and its effects on financial fragility and crisis risk. An important policy conclusion is that redistributive policies can be a useful policy instrument for macroeconomic stability.

The rest of the paper is organized as followed. Section 2 presents a number of key stylized facts. Section 3 presents the model. Section 4 presents model simulations to study the effects of increasing income inequality, and to discuss policy implications. Section 5 concludes.

2. Stylized Facts

This section documents a number of key stylized facts regarding the evolution of the distribution of income, wealth and consumption, changes in household leverage overall and for different groups, the size of the financial sector, and household debt default risk during the financial crisis of 2007-2008. The model presented in the next section will be calibrated to broadly replicate these facts. The appendix documents the data sources used.

Income Inequality and Household Debt: 1929 vs. 2008

Figure 1 plots the evolution of income inequality and household debt ratios in the two decades preceding the two major US crises - 1929 and 2008. In both periods income inequality experienced a sharp increase of similar magnitude: the share of total income commanded by the top 5% of the distribution increased from 24% in 1920 to 34% in 1928, and from 22% in 1983 to 34% in 2007. During the same two periods, the ratio of household debt to GNP or to GDP increased dramatically. It almost doubled between 1920 and 1932, and also between 1983 and 2008, when it reached much higher levels than in 1932. In short the joint evolution of income inequality across high and low income groups on the one hand, and of household leverage on the other hand, displays a remarkably similar pattern in both pre-crisis eras. Our model will replicate this pattern.

Income Inequality and Consumption Inequality

In order to model the consequences of rising income inequality, it is important to clearly document the respective dynamics of income inequality, consumption inequality and wealth inequality. In order to so we use a recent comprehensive dataset compiled by Heathcote, Perri and Violante (2010).\textsuperscript{2}

\textsuperscript{2}The rise in U.S. earnings inequality has been documented since at least Gottshalk and Moffit (1994).
Figure 2, top panel, plots the cumulative percentage changes of male hourly real wages between 1967 and 2005 for three deciles of the distribution of wage earnings: the bottom 10 percentile, the percentile surrounding the median, and the top 10 percentile. Figure 2, bottom panel, plots the cumulative percentage change in real male annual earnings for the same three deciles. Both graphs illustrate the large widening of wage inequality over recent decades. The real hourly wages of the top 10 percentile increased sharply by a cumulative 70%, the real hourly wages around the median declined by 5%, while the wages of the bottom 10% declined strongly, by around 25%. The widening in earnings inequality is even more pronounced when annual earnings are considered reflecting the role of hours and unemployment in the bottom percentile. In the context of our theoretical framework, we take this change in the relative distribution of earnings as the key shock to our model economy, and its calibrated magnitude will be close to the decline observed for the near-median percentile.

Figure 3 documents the evolution of inequality in disposable incomes and in non-durable consumption between 1980 and 2006. The graph plots the ratio of disposable incomes and the ratio of non-durable consumption levels between the top and the bottom 10 percentile of the disposable income distribution. An important finding, already stressed by Slescnik (2001) and Krueger and Perri (2006), is that the rise in income inequality has been much more pronounced than the increase in consumption inequality. The observed magnitudes of income and consumption ratios are approximately reproduced by our model.

**Income Mobility**

To better understand the differences between income inequality and consumption inequality, it is important to assess the importance of intra-generational income mobility. In theory, if increasing income inequality was accompanied by an increase in income mobility, the dispersion in lifetime earnings might be much smaller than the dispersion in annual earnings, as agents move up and down the income ladder throughout their lives. This is a potential explanation for why consumption inequality has been lower than income inequality. However, the data show that, if anything, income mobility has been declining in the United States over the last 40 years, particularly mobility between the top income group and the remainder that we care about in this paper. Bradbury and Katz (2002) document this using the Panel Study of Income Dynamics. For example, in 1969, a family starting in the third quintile of the income distribution had an 18.7% chance of ending up in the top quintile of the distribution by 1979, but for the decade 1988-1998 the same figure falls to 12.8%.
For the same periods, the proportion of families that 10 years later remained in the bottom (top) quintile of the distribution increased from 49.4% (49.1%) to 53.1% (53.2%). Overall, about 40 percent of families ended the 1990s where they began, as compared to 36 percent in the 1970s. These low and falling levels of income mobility mean that income differentials are persistent and translate into unequal lifetime incomes. This in turn provides support for one of our key simplifying modeling choices, the assumption of two income groups with essentially fixed memberships.

**Wealth Inequality and Household Debt Leverage**

In the absence of any change in the valuation of household assets and liabilities, a smaller increase in consumption inequality relative to income inequality must imply that households at the bottom of the distribution of income and wealth are becoming more indebted than households at the top. Figure 4 shows the evolution of the debt to income ratios for different income groups between 1983 and 2007. In 1983, the top wealth groups were somewhat more indebted than the bottom groups, with a gap of around 17 percentage points. In 2007, the relative debt situation has been dramatically reversed: the debt to income ratio of the bottom group, at around 140%, is now twice as high as the debt to income ratio of the top group. Between 1983 and 2007, the debt to income ratios of the bottom groups have more than doubled while the ratio of the top group remains fluctuating around 70 percent. As a consequence almost all of the increase in the debt to income ratio at the aggregate level comes from the bottom group of the wealth distribution. Once again this provides very strong motivation for introducing income heterogeneity in a model of leverage and financial fragility. Our model broadly reproduces the quantitative evolution of the debt to income ratio of the bottom 95%.

**The Size of the US Financial Sector**

In our theoretical framework, the increase in debt of the bottom 95 percent of the distribution generates an increasing need for financial intermediation. Figure 5 plots two measures of the size of the US financial sector between 1980 and 2008. The left panel plots the standard measure of private credit by deposit banks and other financial institutions to GDP. It more than doubled over the period, increasing from 90% in 1981 to 210% in 2008. The right panel plots the share of the financial sector in GDP as constructed by Philippon (2010). According to this measure the financial sector almost doubled in size between 1981 and 2008, and most recently accounted for an extraordinary 8% of U.S. GDP. A similar pattern was again observed prior to the Great Depression.
Leverage, Risk and Financial Crises

As shown in Figure 6, top panel, most of the increase in debt to income ratios for the bottom 95 percent group in the period preceding the crisis was associated with mortgage debt. In the mortgage market, the growing share of subprime loans as documented in Figure 6, bottom panel, is an indicator of the increased riskiness that has accompanied higher leverage. Figure 7 shows evidence of an increase in mortgage debt default risk in 2007-2008 of a magnitude unprecedented since the Great Depression. Default probabilities that increase with leverage, and default rates of the magnitude observed recently, are key ingredients of our model and its calibration.

3. The Model

The model economy consists of two groups of households, referred to as capital owners and workers, and of a production technology that combines the inputs provided by capital owners and workers.

A. Capital Owners

The share of capital owners in the overall population equals $\chi$, which we will calibrate at 5%. They derive utility from consumption and wealth.

Utility from consumption $c_k^t$ has the standard CRRA form, with intertemporal elasticity of substitution $\sigma_k$, but is subject to a subsistence, or minimum acceptable, level of consumption $\bar{c}_k^t$. The subsistence level will be assumed to be either fixed at $c_{k \text{ min}}^t$ or, in one of our scenarios, equal to a fraction of a slow-moving average of past actual consumption levels. The interpretation of subsistence consumption is that most individuals have arranged their affairs in such a manner that a precipitous drop in consumption would represent a catastrophe that is to be avoided at all costs, such as a drastic loss of status or, in the case of workers below, destitution and homelessness.

Wealth in the utility function has been used by a number of authors including Carroll (2000), who refers to it as the “capitalist spirit” specification, Reiter (2004), and Piketty (2010). As explained by the latter, it can represent a number of different saving motives. One is as a reduced form for precautionary savings, because wealth provides security in the presence of uninsurable lifetime shocks. Our preferred interpretation is that agents derive
direct utility from the prestige, power and social status conferred by wealth.\(^3\) Wealth in our
model can take two forms, physical capital held from period \(t\) to \(t + 1\) and denoted by \(k_t\),
and financial investments, or deposits, held from \(t\) to \(t + 1\) and denoted by \(d_t\). Utility from
deposits is assumed to take the log-form that is common in studies of money demand. Utility
from physical capital is assumed to take a Stone-Geary form, with utility derived from the
logarithm of the sum of physical capital, adjusted for expected losses from a crisis event,
and a constant \(\bar{k}\) that determines the sensitivity of desired capital investment to changes
in income. We will study how our results depend on the value taken by \(\bar{k}\). Losses from a
crisis event depend on the probability of a crisis in \(t + 1\), \(\pi_t\), which is taken as given by
households, known by time \(t\), and which will be discussed further below. It also depends on
the percentage of the capital stock destroyed in the event of a crisis, \((1 - \gamma_k)\). The expected
capital stock therefore equals \(k_t (1 - (1 - \gamma_k) \pi_t)\). The same argument applies to the post-
crisis stock of deposits, which equals \(d_t (1 - (1 - \gamma_\ell) \pi_t)\), with \((1 - \gamma_\ell)\) representing
the share of loans defaulted on in a crisis. However, given the logarithmic form of utility from deposits
the expected default loss can be omitted without loss of generality. We therefore have the
lifetime utility function

\[
U^k_0 = E_0 \sum_{t=0}^{\infty} \beta_k^t \left[ \left( \frac{c^k_{t} - \bar{c}^k_{t}}{1 - \frac{1}{\sigma_k}} \right) \left(1 - \frac{1}{\sigma_k}\right) - \xi_d \log (d_t) + \xi_k \log \left( \bar{k} + k_t (1 - (1 - \gamma_k) \pi_t) \right) \right],
\]

where in our baseline

\[
\bar{c}^k_{t} = c^k_{\min},
\]

while we also study the case

\[
\bar{c}^k_{t} = \frac{c^k_{\min}}{\bar{c}^k_{t}} c^k_{t},
\]

with

\[
c^k_{t} = \left( c^k_{t,agg} \left( \bar{c}^k_{t-1} \right)^{\psi} \right)^{1/\psi}.
\]

In the last two expressions \(\bar{c}^k_{t}\) represents the initial steady state level of capital owners’
consumption, \(c^k_{t,agg}\) is the aggregate per capita value of capital owners’ consumption, which
is taken as given by the individual capital owner and which equals \(c^k_{t}\) in equilibrium, and
\(\bar{c}^k_{t}\) is a moving average of past actual capital owners’ consumption, with the parameter \(\psi\)

\(^3\)Carroll (2000) argues that this wealth-loving motive is the best explanation for why saving rates increase
so dramatically with the level of lifetime income. See also Dynan, Skinner and Zeldes (2004) and Kopczuk
(2007).
determining the speed at which the moving average, and therefore the subsistence level, responds to changes in actual consumption.

Capital owners are the owners of the economy’s entire stock of physical capital, whose law of motion is given by

\[ k_t = (1 - \delta) \Delta k_t k_{t-1} + I_t^k. \]  

(5)

Here \( I_t^k \) represents physical investment, and \( \Delta k_t \) equals \( \gamma_k < 1 \) in the event of a crisis, and 1 otherwise. We assume that capital owners do not engage in wage labor, and instead derive all of their income from their ownership of the physical capital stock and from interest on loans to workers. This assumption is made to keep the model parsimonious, but it is not strictly necessary for our main results and could be relaxed to allow for some wage labor in this sector. We let \( q_t \) be the time \( t \) price of a deposit that pays off one unit of output in period \( t + 1 \), \( \Delta \ell_t \) equals \( \gamma_\ell < 1 \) in the event of a crisis and 1 otherwise, and we denote the return to capital \( k_{t-1} \) by \( r_t^k \). Then the capital owner’s budget constraint is given by

\[ d_t q_t = \Delta \ell_t d_{t-1} + r_t^k \Delta k_t k_{t-1} - c_t - I_t^k. \]  

(6)

Capital owners maximize (1) subject to (5) and (6). Letting \( \lambda_t^k \) be the multiplier of the budget constraint, the optimality conditions for consumption, deposits and capital are given by

\[ (c_t^k - \bar{c}_t^k)^{-\frac{1}{\sigma_k}} = \lambda_t^k, \]  

(7)

\[ 1 = \beta_k E_t \left( \frac{\lambda_{t+1}}{\lambda_t^k} \right) \frac{1 - (1 - \gamma_\ell) \pi_t}{q_t} + \frac{\xi_{d}}{\lambda_t^k d_t q_t}, \]  

(8)

\[ 1 = \beta_k E_t \left( \frac{\lambda_{t+1}}{\lambda_t^k} \right) (r_{t+1}^k + 1 - \delta) (1 - (1 - \gamma_k) \pi_t) + \frac{\xi_k}{\lambda_t^k (k + k_t (1 - (1 - \gamma_k) \pi_t))}. \]  

(9)

B. Workers

The share of workers in the overall population equals \( 1 - \chi \), which we will calibrate at 95%. They derive utility from consumption, with the same CRRA form as capital owners’ consumption utility. We use the same notation as for capital owners, with the index \( w \) replacing the index \( k \). Workers inelastically supply one unit of labor per capita. Lifetime utility is given by

\[ U_0^w = E_0 \sum_{t=0}^{\infty} \beta_t^w \frac{(c_t^w - \bar{c}_t^w)^{1 - \frac{1}{\sigma_w}}}{(1 - \frac{1}{\sigma_w})}. \]  

(10)
Workers maximize this utility subject to the budget constraint

$$\ell_t q_t = \Delta \ell_t \ell_{t-1} + c^w_t - w_t,$$

where $\ell_t$ denotes loans obtained from capital owners and $w_t$ is the real wage. Workers default on their loan obligations with a positive probability $\pi_t$ that is increasing in their leverage according to a logistic function. Default events, or financial crises, are accompanied by real crises in which the capital stock is impaired. We will therefore refer to $\pi_t$ not as the default probability but more broadly as the crisis probability. Part of our analysis will consist of experiments that vary the relative sizes of the financial and real components of crises.

The logistic function bounds the crisis probability between 0 and 1, and over the relevant range it implies a crisis probability that is convex in leverage. The leverage that is relevant for the probability of a crisis in period $t+1$ equals the ratio of workers’ loans outstanding at the end of period $t$ to their net income in period $t$, where the latter is defined as their time $t$ wage income minus their net interest obligations on loans outstanding between periods $t$ and $t+1$. We have

$$\pi_t = \frac{\exp \left( \phi_0 + \phi_1 \left( \frac{\ell_t}{w_t - (\frac{1}{\eta t}) \ell_t} \right) \right)}{1 + \exp \left( \phi_0 + \phi_1 \left( \frac{\ell_t}{w_t - (\frac{1}{\eta t}) \ell_t} \right) \right)}.$$  

We adopt this specification in the interest of keeping the model simple and tractable. A relationship between leverage and crisis probability such as (12) arises endogenously in crisis models such as Schneider and Tornell (2004), where a high enough debt leverage moves the economy to a risky zone where a roll-over debt crisis can occur with positive probability.

Workers’ optimality conditions for consumption and loans are given by

$$\left( c^w_t - \tilde{c}^w_t \right) - \frac{1}{\sigma} = \lambda_t^w,$$

$$1 = \beta_w E_t \left( \frac{\lambda_{t+1}^w}{\lambda_t^w} \right) \frac{(1 - (1 - \gamma) \pi_t)}{q_t}.$$

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1 Davig, Leeper and Walker (2010) have, in a different context, adopted an almost identical approach. In their paper the probability of collapse of an initial fiscal regime follows an exogenous logistic function that is increasing in tax rates, and upon collapse the tax rate defaults to an exogenous constant value.
C. Technology

The economy’s aggregate production function is given by
\[ y_t = A \left( \chi \Delta^k k_{t-1} \right)^\alpha (1 - \chi)^{1-\alpha}. \] (15)

Factor returns are determined by the outcome of a Nash bargaining problem over the real wage. Denoting workers’ bargaining power by \( \eta_t \), we have
\[ \max_{w_t} (W_{ht})^{\eta_t} (K_{ht})^{1-\eta_t}, \] (16)
where \( W_{ht} = \lambda_t^w w_t \) is the workers’ surplus, and \( K_{ht} = f_{ht} - w_t \) is the capital owners’ surplus. The marginal product of labor \( f_{ht} \) is in turn given by
\[ f_{ht} = \frac{(1 - \alpha) y_t}{(1 - \chi)}. \] (17)

The first-order condition of the bargaining problem simplifies to
\[ w_t = \eta_t f_{ht}. \] (18)

In other words, the real wage equals workers’ bargaining power times the marginal product of labor. This implies that \( \eta_t \) can fall into the interval \( \eta_t \in [0, \frac{\alpha}{1 - \alpha}] \). The standard competitive outcome obtains at a bargaining power of one. We assume that workers’ bargaining power follows an autoregressive stochastic process that is given by
\[ \eta_t = (1 - \rho) \bar{\eta} + \rho \eta_{t-1} + \epsilon_t^\eta. \] (19)

Finally, the expected rental rate of capital, which enters into the Euler equation for capital (9), is given by
\[ E_t [r_{t+1}^k] = E_t \left[ \frac{A(1 - (1 - \gamma_k) \pi_t) k_t \alpha (1 - \chi)^{1-\alpha} - w_{t+1} (1 - \chi)}{\chi (1 - (1 - \gamma_k) \pi_t) k_t} \right]. \] (20)

D. Equilibrium

In equilibrium capital owners and workers maximize their respective lifetime utilities, and the following market clearing conditions for goods and for financial claims hold:
\[ y_t = \chi (c_t^k + I_t^k) + (1 - \chi) c_t^w, \] (21)
\[ (1 - \chi) l_t = \chi d_t. \] (22)
E. Calibration

Because our study concerns longer-run phenomena, we calibrate the model at the annual frequency. Utility from consumption takes an identical form across agents, with intertemporal elasticities of consumption equal to $\sigma_k = \sigma_w = 0.5$. The subsistence level of consumption equals 50% of initial steady state consumption for the case of fixed subsistence, and 80% for the case of moving-average subsistence. In the latter case the moving-average parameter $\psi$ is set to $\psi = 4$, which implies that the moving average reflects more than 90 percent of any permanent changes in consumption levels within approximately 4 years. The steady state real interest rate $((1/\bar{q}) - 1)$ is fixed at 5% per annum by endogenizing workers’ time preference $\beta_w$. Given the presence of positive capitalist spirit terms in the utility function of capital owners, $\beta_k = 0.9$ is lower than $\beta_w$. The utility weight on financial wealth $\xi_d$ is then determined by imposing an initial steady state loans to workers’ income ratio of 64%, consistent with the U.S. value in 1983. The utility weight on physical capital is determined by imposing an initial steady state gross financial return to capital of 15% per annum, equal to the sum of the real interest rate and the depreciation rate $\delta$, which equals 10% per annum. Finally, the Stone-Geary constant in the utility for physical capital, which affects the elasticity of capital’s response to bargaining power shocks, is set at $\bar{k} = -30$, which compares to a steady state capital stock of 36. There is little guidance in the literature on what the appropriate value for $\bar{k}$ should be, but as we will see a value of zero would be highly inappropriate. We explore the significance of this parameter in Section 4.C.

In the aggregate technology, we normalize steady state output to one through our choice of the parameter $A$. We set the capital share parameter equal to $\alpha = 0.27$, which generates a steady state investment to GDP ratio of 18%, consistent with U.S. data. It also implies an initial steady state income share of capital owners of 29.8%, which in the data compares to 23% in the early 1980s and 34% in recent times. The standard deviation of bargaining power shocks is assumed to equal $\sigma_\eta = 0.015$, and the mean bargaining power $\bar{\eta} = 1$ replicates the competitive outcome.

A crisis event is characterized by the probability of its occurrence, and by the size of the collapses in loans and capital, and therefore in output, if it does occur. We set the two coefficients of the logistic function to $\phi_0 = -7.5$ and $\phi_1 = 3$. As illustrated in Figure 8, this produces a baseline crisis probability of 0.38% at a leverage of 64%, and a convex relationship between leverage and the crisis probability that reaches around 5% at a leverage of 150%. This range is consistent with the probability of major disaster events estimated by Barro
Next we calibrate the size of disaster events, that is of major defaults on loans and of output collapses. Based on International Monetary Fund (2009), the reductions in the level of output associated with major financial crises that happened simultaneously with real crises have averaged 3.4%. We generate a comparable output collapse by assuming capital destruction in the event of a crisis equal to 10% of the pre-existing capital stock, $\gamma_k = 0.9$. Given the capital share parameter in the technology this leads to an output collapse of around 2.7%. While this is less than 3.4%, it may nevertheless be too high, given that our model does not feature endogenous labor supply and therefore a contribution to output collapse from higher unemployment. To test the sensitivity of our results to the assumption of $\gamma_k = 0.9$ we will also explore an alternative scenario where the capital destruction only equals 1%, or $\gamma_k = 0.99$. The percentage of loans defaulted upon during the crisis is based on the U.S. experience, up to this point, with the financial crisis that started in 2008. This crisis has seen mortgage past due rates approaching 10%. We therefore set $\gamma_\ell = 0.9$.

**F. Solution Methods**

The above model has two features that make it unsuitable for the application of conventional perturbation methods. The first is the presence of large and discrete crisis events, which under our calibration imply jumps in state variables of up to 10 percent. The second is the fact that the model’s two endogenous state variables, capital and loans, are extremely persistent, and are then subjected to large bargaining power shocks, which means that they can drift far away from their original steady state for a very long period. It is therefore necessary to apply global solution methods. We adopt and compare two different approaches.

First, the baseline version of our model has three continuous state variables (capital, loans and bargaining power) and one binary state variable (crisis or no crisis). This is sufficiently tractable to permit the use of functional iteration on a discretized state space to compute solutions. Specifically, we use the monotone map method of Coleman (1991), which has

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5Applied to the 2008 crisis this quite low perceived probability seems appropriate given the evident surprise of a majority of commentators at the outbreak of the crisis. It is a separate question whether this assessment was realistic, given the historically unprecedented household leverage ratios in 2008, even when compared to the Great Depression.
recently been used in a number of papers by Davig, Leeper and Walker. The monotone map method discretizes the state space and finds a fixed point in decision rules for each grid point in the state space. It substitutes a set of conjectured decision rules into the model’s intertemporal Euler equations, and iterates until the iteration improves the current decision rule at any given state vector by less than some $\varepsilon$. As initial conjectures we use decision rules computed by DYNARE for a first-order approximation of the model. These conjectures are applied to a version of the nonlinear model with only a small fraction of the full standard deviation $\sigma_\eta$, and with a narrow grid for the state space, based on the conjecture that for a sufficiently small standard deviation the solutions are approximately linear. Both the standard deviation and the grid width are then sequentially increased, and at each step the results of the previous iteration, appropriately scaled up or down to account for the wider spacing of grid points, are used as initial guesses. Numerical integration is used to compute expectations. As evidence of local uniqueness, we perturb the converged decision rules in various dimensions and check that the algorithm converges back to the same solution.

We present 50-year impulse responses for a standardized realization of bargaining power and crisis shocks, namely an initial decline in workers’ bargaining power from $\bar{\eta} = 1$ over a period of 10 years, followed by a very gradual return to $\eta = 1$, and a crisis event in year 30. This can be thought of as a highly stylized representation of the events preceding either 1929 or 2008. Sensitivity analysis varies a number of aspects of this shock sequence, including the size of the decline in bargaining power over the first 10 years, the speed of reversal to $\eta = 1$ after year 10, the size of the crisis event, the elasticity of capital accumulation with respect to bargaining power shocks, the the form (fixed or variable) of subsistence consumption.

The version of our model with moving-average subsistence levels of consumption has five continuous and one binary state variable, which makes application of the monotone map method computationally very costly. We therefore also use a second solution method, a perfect foresight solution in TROLL using a Newton-based stacking algorithm. Because a comparison of perfect foresight simulations with monotone map simulations also yields interesting additional insights for the simulations that assume a fixed subsistence level of consumption, we present simulation results using this algorithm for all impulse responses reported in the paper. In this case the probability of a crisis event enters optimality conditions in the same way as before, but the bargaining power shocks hitting the economy over

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the first 10 years are unanticipated, and the subsequent evolution of bargaining power is expected with certainty. Specifically, the entire infinite horizon economy is simulated for the first year assuming only the first year’s shock, which is then repeated for the second year taking as given the state variables inherited from the first year, and so on until year 10, after which no further bargaining power shocks are expected to hit, and slow convergence back to $\eta = 1$ occurs, at a rate determined by $\rho$. In period 30, the time of the crisis event, a final infinite horizon simulation, taking as given the values of the state variables, $\gamma_k k_{29}$ and $\gamma_\ell \ell_{29}$, is performed.

4. Simulated Scenarios

Figures 9-14 present a baseline simulation and a number of alternatives that explore the sensitivity of our main conclusions to the calibration of the model. The simulations are highly stylized, in that while there has certainly been a decline in bargaining power for lower income groups in the United States, we do not claim to have reproduced its precise intertemporal pattern. Rather, we trace its broad implications, and their dependence on key structural characteristics of the economy. In each case the perfect foresight simulation is shown as a black solid line, and the monotone map simulation as a red line with red markers. The horizontal axis represents time, with the shock hitting in year 1 and the final period shown being year 50. Simulations are initiated, both under perfect foresight and under uncertainty, at the steady state vector of the deterministic steady state (more on this below). The vertical axis shows percent deviations from the initial deterministic steady state for real stock and flow variables, percentage point deviations for rates of return, percentage points for leverage, crisis probability, the interest expense to income ratio, and the income and consumption shares of capital owners, and simple ratios for the relative per capita income and consumption levels of capital owners and workers.

A. Baseline Scenario

Figure 9 presents our baseline scenario, with a cumulative 7.5% decline in workers’ bargaining power over the first 10 years$^8$, followed by a very slow reversal back to $\eta = 1$ determined by the autogressive parameter $\rho = 0.96$. The crisis event happens in year 30, and features 10% collapses in loans and capital, $\gamma_\ell = \gamma_k = 0.9$.

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$^8$This corresponds to a shock of one half of one standard deviation in each year.
Apart from some important details that we will discuss in the next subsection, the monotone map and perfect foresight simulation results are very similar. The real wage over the initial decade collapses by close to 6 percent, while the return to capital increases by over 2 percentage points. Workers’ consumption however declines by only around two thirds of the decline in wage income, as workers borrow the shortfall from capital owners, who have surplus funds to invest following their increase in bargaining power. Over the 30 years prior to the outbreak of the crisis, loans more than double to bring workers’ leverage from 64 percent to around 140 percent, with the crisis probability in year 30 exceeding 3 percent. The loan interest rate for most of this initial period is up to 2 percentage points above its initial value, as lenders arbitrage the return to lending with the now higher return to capital investment.

Capital owners’ share of the economy’s income increases from initially less than 30 percent to over 35 percent. They have three ways to dispose of the extra income, and they utilize all three in a way that equalizes their marginal contributions to utility. First, their consumption increases by eventually over 20 percent prior to the outbreak of the crisis. Second, capital investment increases by over 15 percent, and so does the physical capital stock. The increase in capital raises the economy’s output by eventually close to 4 percent. And third, loans increase by over 100 percent, which means that capital owners’ consumption share increases by only around 2 percentage points, compared to 5 percentage points for their income share. These last two points are closely related, because with 71 percent of the economy’s final demand coming from workers’ consumption, this output cannot be sold unless a significant share of the additional income accruing to capital owners is recycled back to workers by way of loans. With workers’ bargaining power, and therefore their ability to service and repay loans, only recovering very gradually, the increase in loans is extremely persistent.

The initial gain in capital owners’ rate of return of more than 2 percentage points is thereafter pared back by two factors. First, the large increase in investment reduces the marginal product of capital, and second, the gradual return of workers’ bargaining power increases their wage and thus reduces what is left for capital. By year 30 profitability has in fact declined below its initial level. At that point there are two ways to again raise the return to capital. One would be another round of increasing capital owners’ bargaining power. And the other is a major crisis that destroys large amounts of existing capital. We observe the latter in year 30, but the respite for capital owners is only temporary in the presence of the ongoing recovery in workers’ bargaining power. Unless this changes, the inevitable result
will be a prolonged period of low profitability, in the sense of rates of return that remain below those in the initial steady state.

We interpret the crisis as a release of the increasing pressure built up on workers’ balance sheets, with the interest portion of debt service increasing from initially around 3 percent to 6 percent of their income at the time of the crisis, and prospects for an early reduction in leverage very low given the slow recovery in bargaining power. The crisis however barely improves workers’ situation. While their loans drop by 10 percent due to default, their wage also drops significantly due to the collapse of the real economy, and furthermore the real interest rate on the remaining debt shoots up to raise debt servicing costs to 9 percent of income. As a result their leverage ratio barely moves, and for the present calibration it in fact increases further later on so that by year 50 it is above its pre-crisis level, with a very slow reduction thereafter. It is however clear that this last result depends critically on the relative sizes of the loan default versus the collapse in the real economy. As we will see below, when the crisis mainly affects loans, it does bring more significant relief to workers.

B. Uncertainty

The simulations based on the monotone map method, which take uncertainty concerning future levels of bargaining power into account, show a number of interesting differences to the perfect foresight case.

One is that at the outset capital owners briefly but sharply reduce consumption to permit a boost in capital investment, thereby supporting a faster increase in the capital stock. Loans also initially increase at a faster rate. The reason is that we have initialized both simulations at the state vector of the deterministic steady state. Under uncertainty however, capital owners would prefer higher capital and loan stocks even in the absence of realized negative shocks to $\eta$. This is because volatile bargaining power, by affecting incomes, increases consumption risk and thus lowers the expected utility of consumption. Capital owners can reduce their exposure to that risk by switching from consumption to holdings of capital and loans, which also offer utility but which are not equally affected by changes in bargaining power. In our baseline simulation the long-run value for workers’ leverage is therefore around 90 percent rather than 64 percent, and around a third of the increase in leverage observed over the pre-crisis period is due to convergence to this higher long-run value, with the other two thirds accounted for by the realized shocks to $\eta$. The relative effects of uncertainty versus realized $\eta$ on the capital stock are similar. Putting this differently, if our simulations under
uncertainty were initialized at the stochastic rather than the deterministic steady state, the effects of realized bargaining power shocks on leverage and the capital stock over the first 30 years would be relatively smaller, but still very large in absolute terms.

Another interesting difference between the uncertainty and perfect foresight simulations concerns the longer-run behavior of capital and especially loans, which under uncertainty are noticeably lower at the 50-year horizon. The reason is that, at the very high levels of debt and capital reached by that time, the convexity of the crisis probability function assumes increasing importance. It implies that under uncertainty about future bargaining power the expected probability of a crisis is significantly higher, and therefore the willingness of capital owners to be exposed to such a crisis, through high stocks of loans and capital, is significantly lower. Of course in the very long run this picture is again reversed, as the perfect foresight economy returns to a leverage of 64 percent, while the economy under uncertainty settles at a leverage of around 90 percent.

C. High Leverage - Aggravating Factors

The baseline scenario has leverage increasing to around 135 percent by the time of the crisis (125 percent under uncertainty), and remaining in the neighborhood of that value for decades afterwards, with a crisis probability hovering in the neighborhood of 3.5 percent for several decades (2 percent under uncertainty). This outcome however depends on a number of aspects of the calibration of the model and of the specification of shocks, and changes to these can make the outcome for leverage worse or better. We begin by describing the factors aggravating crisis risks in this subsection, and in the next subsection we turn to possibilities for bringing down leverage.

In the baseline workers are partly compensated for their loss of bargaining power by the fact that capital owners invest part of their additional income in physical capital, which over time helps to raise the real wage. Figure 10 considers an alternative calibration where the marginal benefit to capital owners of doing so is reduced, so that more of their gains from higher bargaining power are either consumed or invested in loans. Specifically, by setting $\tilde{k} = -33$ instead of $\tilde{k} = -30$, capital accumulation is reduced by one third over the first 30 years, and output growth is reduced accordingly.9 One result is a further 2 percentage point increase in the consumption share of capital owners, as they consume instead of investing.

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9It can therefore be seen that setting $\tilde{k}$ much closer to zero would imply a massive and clearly implausible response of capital accumulation to income shocks.
The other is that leverage now reaches around 145 percent by the time of the crisis, and thereafter keeps growing to 175 percent by year 50 under perfect foresight, while it stays near 135 percent under uncertainty. Furthermore, the crisis itself is now characterized by a small increase rather than a decrease in leverage and in crisis probability. The longer-run crisis probabilities (almost 8 percent by year 50 under perfect foresight, 3 percent under uncertainty) are far higher than in the baseline. The use by capital owners of the extra funds gained at the expense of workers is therefore a critical determinant of the sustainability of lower worker bargaining power. If a large share of the funds is invested productively, higher debt is more sustainable because it is supported by higher income. If instead the majority of the funds goes into capital owners’ consumption, or into loan growth, in other words an increasing “financialization” of the economy, the system becomes increasingly unstable and prone to crises.

A second aspect of the baseline calibration that might be too optimistic is the rate at which workers’ bargaining power is restored, after the initial period of declining bargaining power of 10 years. With \( \rho = 0.96 \), 50 percent of the loss of bargaining power is reversed by year 27. This was not an obvious feature of the pre-1929 and pre-2008 periods. Figure 11 therefore considers an alternative scenario with \( \rho = 0.99 \), which is close to permanent, with the half-life of bargaining power equal to 80 years instead of 27 years. In this case the initial loss of bargaining power is assumed to be smaller, with \( \eta \) dropping to 0.95 by year 10, rather than to 0.925 as in the baseline. Given the smaller initial drop in \( \eta \), the increase in leverage and crisis probability by year 30 is of course smaller. But more interesting for our purposes is the fact that thereafter leverage keeps increasing further, including under uncertainty, and the crisis probability keeps climbing. It can in fact be shown that for this scenario the crisis probability does not peak until 50 years after the first crisis under uncertainty, and another 30 years later under perfect foresight. This illustrates a key concern. If workers see virtually no prospects of restoring their earnings potential even in the very long run, high leverage and high crisis risk become an almost permanent feature of the economy.

The third modification of the baseline that can give rise to higher crisis risk is a higher subsistence level of consumption. For most households it probably takes far less than a halving of consumption levels to arrive at what they perceive to be a catastrophic event. A large number of households in modern economies, and not only the relatively poor, does in fact live paycheck to paycheck and would have to radically rearrange their affairs if faced with
even a small drop in income. The scenario in Figure 12 therefore raises the subsistence consumption level to 80 percent of initial steady state consumption, but allows for that subsistence level to change gradually over time in response to realized consumption levels. We observe that under this specification households borrow much more aggressively than in the baseline to avoid a drop in consumption. As a result leverage reaches 155 percent at the time of the crisis, and close to 170 percent around year 40, with a crisis probability that reaches 8 percent at its peak. However, under this specification workers are eventually willing to significantly reduce consumption, as their subsistence level comes down in the light of a prolonged experience of low consumption. Over the longer run this stabilizes leverage and avoids near explosive debt. A moving average parameter $\psi$ that allows for a more rapid adjustment of subsistence consumption would strengthen this effect, but $\psi$ is already at a level where subsistence adjusts fairly rapidly.

D. High Leverage - Solutions

The currently much talked about deleveraging of households can in the present model take only two forms, a reduction in debt levels, either through default or through debt forgiveness, or an increase in workers’ earnings to allow them to work their way out of debt over time. We address each of these in this subsection.

An orderly debt reduction that does not cause upheaval and large output losses in the real economy reduces leverage much more powerfully than in the baseline. Figure 13 illustrates the case where the destruction of physical capital at crisis time only equals 1 percent instead of 10 percent, leaving all other aspects of the baseline calibration unchanged. The main difference to the baseline is that in this case the debt reduction is not accompanied by a significant income reduction, as the real wage drops very little. As a result, leverage drops by 13.5 percentage points, compared to 3 percentage points in the baseline. Minimizing spillovers from the financial to the real sector during a widespread debt restructuring to deal with excessive leverage is therefore critical to the success of that restructuring.

Figure 14 illustrates the alternative to a debt restructuring, an increase in workers’ earnings through a restoration of their original bargaining power. In this case the evolution of the economy is identical to the baseline until period 30, but at that time a program is implemented whereby workers’ bargaining power immediately and permanently returns to

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10In a recent survey by the largest U.S. employment website (CareerBuilder (2010)), 77 percent of respondents report that they live paycheck to paycheck, up from 61 percent in 2009.
\[ \eta = 1. \] The assumption is that this is sufficient to head off a crisis event. We do not take a stance on how such a policy could be implemented, instead we only trace its macroeconomic implications. The first result is an upward jump in the real wage to about 4 percent above its value in period 0, due to the now much higher capital stock. Leverage drops by 8 percentage points on impact (both under perfect foresight and under uncertainty), but this is now not due to a lower, restructured loan stock, but rather to a higher income level, which is of course helped by the fact that this turn of events is assumed to head off a collapse in capital and output. The main difference to Figure 13 however is observed following period 30, where under a loan restructuring leverage and default probability resume an upward trajectory for several additional decades, while under the bargaining power solution both immediately go onto a declining path. By year 50 leverage is around 20 percentage points lower under the bargaining power solution than under the loan restructuring solution. For long-run sustainability a flow adjustment, giving workers the means to repay their obligations over time, is therefore much more successful than a stock adjustment, unless the latter is extremely large.

5. Conclusions

The paper has presented stylized facts and a theoretical framework that explore the nexus between increases in the income advantage enjoyed by high income households, higher debt leverage among poor and middle income households, and vulnerability to financial crises. This nexus was prominent prior to both the Great Depression and the recent crisis. In our model it arises as a result of increases in the bargaining power of high income households. The key mechanism, reflected in a rapid growth in the size of the financial sector, is the recycling of some of the additional income gained by high income households back to the rest of the population by way of loans, thereby allowing the latter to sustain consumption levels, at least for a while. But without the prospect of a recovery in the incomes of poor and middle income households over a reasonable time horizon, the inevitable result is that loans keep growing, and therefore so does leverage and the probability of a major crisis that, in the real world, typically also has severe implications for the real economy. More importantly, unless loan defaults in a crisis are extremely large by historical standards, and unless the accompanying real contraction is very small, the effect on leverage and therefore on the probability of a further crisis is quite limited. By contrast, restoration of poor and middle income households’ bargaining power can be very effective, leading to the prospect of a sustained reduction in leverage that should reduce the probability of a further crisis.
The framework we have presented uses a closed economy setting. In future work we aim to extend this to an open economy. It is clear that the same mechanism presented in this paper, namely the increase in lending by high income households in the country that is subject to a bargaining power shock favoring high income households, would then extend not just to domestic poor and middle income households, but also to foreign households. The counterpart of this capital account surplus in the foreign country would of course be an increase in its current account deficit. In other words, this provides a potential mechanism to explain global current account imbalances triggered by increasing income inequality in surplus countries.

References


Data Appendix

Here we briefly describe the sources for the data used to produce Figures 1-7. The data in Figure 1, top panel, come from several issues, from the 1920s and 1930s, of the U.S. Department of Commerce’s Statistical Abstract of the United States. In Figure 1, bottom panel, data for income shares of the top 5% income bracket are taken from the updated dataset of Piketty and Saez (2003), while data for household debt to income ratios come from the Federal Reserve Board’s Flows of Funds database. Figures 2 and 3 use a dataset compiled by Heathcote, Perri and Violante (2010). The reader is referred to that paper for information on the underlying data sources. Figure 4 uses data computed from various issues of the triennial Survey of Consumer Finances between 1983 and 2007. Figure 5, top panel, shows a measure of real private credit by deposit banks and other financial institutions, relative to GDP, from the World Bank Financial Structure Database\(^\text{11}\). Figure 5, bottom panel, presents information on the value added share of the financial sector in U.S. GDP, as computed by Philippon (2008). Figure 6, top panel, shows data on mortgage debt to income ratios computed from the Survey of Consumer Finances. Figure 6, bottom panel, shows data from the Mortgage Banker Association on the partition of the stock of mortgages serviced in the United States between prime and subprime borrowers. Figure 7 uses data form a number of Mortgage Banker Association Delinquency Surveys, and assembled by Haver Analytics.

\(^{11}\)See http://econ.worldbank.org/
Figure 1: Income Inequality and Household Leverage.
Figure 2: Real Earnings Inequality.
Figure 3: Income Inequality and Consumption Inequality.

Figure 4: Debt to Income Ratios
Figure 5: The Size of the U.S. Financial Sector.

Figure 6: Mortgage Debt and Subprime Borrowing.
Figure 7: Mortgage Default. Share of Past Due Loans.

Figure 8: Leverage and Crisis Probability in the Model
Figure 9: Baseline Scenario
Figure 10: Less Capital Investment ($\bar{k} = -33$)
Figure 11: Nearly Permanent Change in Bargaining Power ($\rho = 0.99$)
Figure 12: High Variable instead of Low Fixed Subsistence Consumption
Figure 13: Orderly Debt Restructuring ($\gamma_k = 0.99$)
Figure 14: Restoration of Workers’ Bargaining Power ($\eta_{30^+} = 1$)