Trade, firm selection, and innovation: the competition channel*

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Abstract

The availability of rich firm-level data sets has recently led researchers to uncover new evidence on the effects of trade liberalization. First, trade openness forces the least productive firms to exit the market. Secondly, it induces surviving firms to increase their innovation efforts and thirdly, it increases the degree of product market competition. In this paper we propose a model aimed at providing a coherent interpretation of these findings. We introducing firm heterogeneity into an innovation-driven growth model, where incumbent firms operating in oligopolistic industries perform cost-reducing innovations. In this framework, trade liberalization leads to higher product market competition, lower markups and higher quantity produced. These changes in markups and quantities, in turn, promote innovation and productivity growth through a direct competition effect, based on the increase in the size of the market, and a selection effect, produced by the reallocation of resources towards more productive firms. Calibrated to match US aggregate and firm-level statistics, the model predicts that a 10 percent reduction in variable trade costs reduces markups by 1.15 percent, firm surviving probabilities by 1 percent, and induces an increase in productivity growth of about 13 percent. More than 90 percent of the trade-induced growth increase can be attributed to the selection effect.

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1 Introduction

Following the observation that large and persistent productivity differences exist among firms within the same industry (e.g. Bartelsman and Doms, 2000), an interesting set of empirical regularities about international trade has recently emerged from a large numbers of studies using firm-level data. Firstly, strong evidence suggests that trade liberalization induces the least productive firms to exit the market, reallocating both demand and resources to surviving, more productive firms; this is the so-called selection effect of trade resulting in an increase of aggregate productivity (see e.g. Pavcnik, 2002, Topalova, 2004, and Tybout, 2003 for a survey).

A second line of research has focused on the innovation effect of trade, by documenting the role of firm heterogeneity in shaping the effects of trade liberalization on innovation activities and productivity growth. Bustos (2010) shows that a regional trade agreement, Mercosur, has selected highly productive firms into exporting and affected positively a broad set of innovation measures (computers and software, technology transfers, R&D, and patents).\(^1\) Bloom, Draca, and Van Reenen (2009) study the effect of Chinese import penetration on innovation in European countries. They find evidence of both the selection and the innovation effect of trade. On the one hand Chinese competition decreases employment and firm’s chances of survival, and this effect is stronger for low-tech than for high-tech firms. On the other hand, surviving firms tend to innovate more (patenting and R&D) and upgrade their technology (IT intensity). Aw, Roberts, and Xu (2010) using plant-level Taiwanese data estimate a structural model of firm’s decision to invest in R&D and enter the export market. They find that a reduction in trade costs produces a substantial increase in innovation and export, both resulting in a fairly persistent increase in productivity growth.\(^2\)

A third piece of evidence shows that trade liberalization has pro-competitive effects that can potentially lead to more selection and more innovation. Bugamelli, Fabiani, and Sette (2008) using Italian firm-level manufacturing data find that import competition from China has reduced prices and markups in the period 1990-2004. Griffith, Harrison, and Simpson (2008) have studied the effects of trade integration reforms carried out under the EU Single Market Programme and found that these reforms have increased product market competition (measured as average markups) and stimulated innovation (R&D expenditures). Chen, Imbs and Scott (2008) and

\(^1\)Focusing on innovation has the advantage of identifying one specific channel through which improvements in productivity take place. Other studies have instead estimated productivity as a residual in the production function, with the consequence that together with technological differences, residuals captures also other differences such as market power, factor market distortion, and change in the product mix. (see i.e. Foster, Hallowanger, and Syverson, 2008, Hsieh and Klenow, 2008, and Bernard, Redding, and Schott, 2008).

\(^2\)Several papers have investigated the related but slightly different question of whether the exporter status implies a higher investment in innovation or technology upgrading: this has been called the learning by exporting mechanism. The evidence is mixed: early papers, such as Clerides, Lach, and Tybout (1998) and Bernard and Jensen (1999) do not find any evidence in favor of this mechanism. Recent studies have instead found evidence that firms improve their productivity subsequent to entry (e.g. Delgado, Farinas, and Ruano, 2002, De Loecker, 2006, Van Biesbroek, 2005, see Lopez, 2005, for a survey). The basic difference between these studies and those discussed in the main text is that the former focus on productivity and the latter on innovation. One exception is Cruiscolo, Haskel, and Slaugther (2008) which finds that exporters and multinational firms have higher productivity because they both innovate more and learn from foreign technologies. The other difference is that Bustos (2010), Bloom et al (2008), and Lileiva and Treffer (2008) focus on trade liberalization and not on export status.
Corcos, Del Gatto, Ottaviano and Mion (2010) using micro data on EU countries estimate the Melitz and Ottaviano (2008) model and show that trade openness reduces average prices and markups, while raising productivity through firm selection.

This paper presents a theoretical model aimed at providing a coherent interpretation of these empirical findings. More precisely, we set up a model in which trade liberalization has pro-competitive effects, through reduced markups, leading to firm selection and increased innovation. A dynamic industry model with heterogeneous firms is added to a growth model with innovation by incumbents. There are two goods, a homogeneous good produced under constant returns, and a differentiated good produced with a continuum of varieties, each of them facing both variable and fixed production costs. As in Hopenhayn (1992) and Melitz (2003), productivity differs across varieties. Firms in the differentiated good sector allocate labor to the production of a specific variety and to innovation activities aimed at reducing their production costs. Each variety is produced by a small number of identical firms, operating in an oligopolistic market; thus quantities produced and innovation activities result both from the strategic interaction among firms. The oligopolistic market structure and the cost-reducing innovation features are borrowed from static trade models with endogenous market structure (e.g. Neary, 2009 and 2010, Eckel and Neary, 2010) and from multi-country growth models with representative firms (e.g. Peretto, 2003 and Licandro and Navas 2007) respectively.

The open economy features two symmetric countries engaging in costly trade (iceberg type). In order to simplify the analysis, the baseline version of the model assumes no entry costs into the export market, implying that all operating firms export, and takes the number of oligopolistic producers as given. In an extended version of the model, we remove these assumptions and show that the main mechanisms behind our results are still at work. When the economy moves from autarky to free trade, product market competition increases since the number of firms operating in each local market doubles. This produces a reduction in the markup and a decrease in the inefficiency of oligopolistic markets, ultimately leading to an expansion of the quantities produced by firms. Since innovation is cost-reducing, the trade-induced increase in the market size increases firms’ incentive to innovate. We call this the direct competition effect. Moreover, a decline in the markup raises the productivity cutoff and moves the least productive firms out of the market. This selection effect reallocates resources from exiting to surviving firms, increasing their average size and their incentive to invest in cost-reducing innovation. Hence, trade-induced firm selection increases not only the ‘level’ of aggregate productivity (as in Melitz, 2003) but also firms’ innovation, affecting the ‘growth rate’ of productivity as well. We call this new mechanism the selection effect of competition. Incremental trade liberalization (reduction in the iceberg trade cost) has similar effects. The direct effect can be obtained even in a model with representative firms, as shown in Peretto (2003) and Licandro and Navas (2007), while the new channel highlighted in this paper is strictly dependent on the presence of heterogeneous firms.

Finally, the baseline model is calibrated to match salient firm-level and aggregate statistics of the US economy, showing a sufficiently good fit of the data. It shows that a reduction in trade
costs has a quantitatively relevant selection effect of competition on innovation. Precisely, a 10 percent reduction in trade costs increases the aggregate growth rate by about 13 percent, a big share of which (about 96 percent) comes from selection and the rest from the direct competition effect. Extensive sensitivity analysis shows that this growth decomposition is fairly robust to changes in parameters value, with the selection effect systematically accounting for more than 90 percent of the overall growth effect of trade. This suggests that the selection channel, which represents the main innovation of our paper is quantitatively relevant.

This paper is related to the emerging literature on the joint effect of trade liberalization on selection and innovation. A first line of research introduces a one-step technological upgrading choice into a heterogeneous firm framework. Examples are Yeaple (2005), Costantini and Melitz (2007), Bustos (2010), Navas and Sala (2007) and Vannooerenberge (2008). In all these papers, with the exception of Costantini and Melitz, the model economy is static. Our paper is more closely related to a second stream of research that introduces innovation as a continuous process in dynamic models of trade and productivity growth. Baldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2008) explore the effects of trade liberalization on innovation and growth in models of expanding product variety (Romer, 1990) with heterogeneous firms. They show that the effect of trade-induced firm selection on innovation and growth depends on the form of (international) knowledge spillovers characterizing the innovation technology. Atkeson and Burnstein (2010) set up a model of process and product innovation with firm heterogeneity and show that trade has positive effects on process innovation that can be offset by negative effects on product innovation. Although differing in the innovation type or in the innovation technology they analyze, all these papers adopt a monopolistically competitive market structure.

The key distinguishing feature of our model is that it focuses on the interactions between trade, firm heterogeneity and innovation in an dynamic oligopolistic environment. In this framework, the market structure is endogenous and responds to changes in trade costs, thereby representing the ideal environment to analyze the effects of trade on product market competition (the third stylized fact discussed above). Melitz and Ottaviano (2008) show that under a particular form of preferences it is possible to obtain endogenous markups in the monopolistic competitive framework. In line with our result, they find that trade liberalization produces a pro-competitive effect (lower markups) and triggers the selection of the least productive firms out of the market.

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4One exception is Van Long, Raff and Stahler (2008) that features an oligopolistic market structure, but innovation is not a continuous process and the model is static.

5The presence of endogenous markups allows the selection effect to work through a channel different from that highlighted in Melitz (2003). In that paper, trade liberalization produces an increase in labor demand that bids up wages and forces low productivity firms to exit. In our paper, as in Melitz and Ottaviano (2008), the selection effect is produced by the reduction in markups brought about by trade liberalization. While there is evidence, as discussed above, that trade liberalization has increased product market competition, the trade-induced increase in average wages triggering firm selection in Melitz (2003) seems less in line with the data. For instance March
Our model differs from that of Melitz and Ottaviano not only for the different source of endogenous markups but also because in their model there is no innovation activity aimed at improving productivity, therefore they cannot study the implications of firm heterogeneity and endogenous markups for innovation. Bernard, Jensen, Eaton, and Kortum (2003), set up a Ricardian model with Bertrand competition among firms and obtain markups responding endogenously to trade liberalization. We complement their analysis by introducing innovation and deriving endogenous markups from Cournot competition. Finally, we contribute to the literature on oligopoly and trade (e.g. Neary, 2003, Eckel and Neary, 2009, Venables, 1985, and Horstman and Markusen, 1986) introducing heterogeneous firms and innovation-driven growth.

Summing up, to our knowledge the present paper is the first to provide a framework to interpret jointly the three stylized facts discussed above. The basic structure of the model is such that trade affects both firm selection and innovation through the competition channel, that is through its effect on the markup. The selection effect of trade operating through endogenous markups resulting from oligopolistic competition among firms is a novel contribution. Secondly, while the direct competition effect of trade on innovation is not new in the literature (see Peretto, 2003, and Licandro and Navas, 2007), the interaction between firm selection and innovation represents an original contribution of this paper.

2 The model

2.1 Economic environment

The economy is populated by a continuum of identical consumers of measure one. Time is continuous and denoted by $t$, with initial time $t = 0$. Preferences of the representative consumer are

$$
\int_0^\infty (\ln X_t + \beta \ln Y_t) e^{-\rho t} \, dt,
$$

with discount factor $\rho > 0$. There are two types of goods: a homogeneous good, taken as the numeraire, and a differentiated good. Consumers are endowed with a unit flow of labor, which can be transformed one-to-one into the homogeneous good. This implies that equilibrium wages are equal to one. The amount $Y$ of the labor endowment is allocated to homogeneous good production, which enters utility with weight $\beta > 0$.

The differentiated good $X$ is produced with a continuum of varieties of endogenous mass $M_t \in [0, 1]$, according to

$$
X_t = \left( \int_0^{M_t} x_{jt}^\alpha \, dj \right)^{\frac{1}{\alpha}}, \tag{1}
$$

where $x_{jt}$ represents variety $j$, and $1/(1-\alpha)$ is the elasticity of substitution across varieties, with $\alpha \in (0, 1)$. Each variety in $X$ is produced by $n$ identical firms using labor to cover a fixed

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CPS data show that both median and average US wages have stagnated in the last three decades, a period of progressive trade liberalization (see Acemoglu, 2002)
production cost $\lambda > 0$ and variable cost. The variable cost, and hence firm productivity, differ across varieties. A firm with productivity $\tilde{z}_t$ has the following production technology

$$\tilde{z}_t^{-\eta} q_t + \lambda = y_t,$$

where $y$ represent inputs and $q$ production (we omit index $j$ and identify the variety with its productivity). Variable costs are assumed to be decreasing in the firm’s state of technology with $\eta > 0$.

Innovation activities are undertaken by incumbents according to the following technology

$$\tilde{z}_t = A \tilde{z}_t h_t,$$

where $h$ represents labor allocated to innovation and $A > 0$ is an efficiency parameter. An externality $\tilde{z}$, which will be defined later, affects the productivity of the innovation technology. We assume, for simplicity, that all firms producing the same good have the same initial productivity $\tilde{z}_0 > 0$.

Irrespective of their productivity, varieties exit the market at rate $\delta > 0$. Exiting varieties are replaced by new varieties in order for the mass of operative varieties to remain constant at the steady-state equilibrium. Below we derive the equilibrium, restricting the analysis to the steady state.

### 2.2 Households

The representative household maximizes utility subject to its instantaneous budget constraint. The corresponding first order conditions are

$$Y = \beta E,$$

$$\frac{\dot{E}}{E} = r - \rho = 0,$$

$$p_{jt} = \frac{E}{X^a_{jt}} X_j^{a-1},$$

where $r$ is the interest rate and $p_{jt}$ is the price of good $j$. Total household expenditure on the composite good $X$ is

$$E = \int_0^M p_{jt} x_{jt} \, dj.$$

Because of log preferences, total spending in the homogeneous good is $\beta$ times total spending in the differentiated good. Equation (5) is the standard Euler equation implying $r = \rho$ at the stationary equilibrium, and (6) is the inverse demand function for variety $j \in [0, 1]$. Variables $Y, E, M$ are also constant at steady state (index $t$ is then omitted to simplify notation).

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6Perfect substitution is implicitly assumed among the $n$ goods belonging to a particular variety. In a more general framework, the degree of substitution across these $n$ goods may be finite even if it must be larger than the degree of substitution across varieties. Introducing another degree of imperfect substitutability across goods would complicate notation without adding any key insights.
2.3 Production and Innovation

Firms producing the same variety behave non-cooperatively and maximize the expected present value of their net cash flow:

\[ V_{ijs} = \int_{s}^{\infty} \pi_{ijt} e^{-\left(\rho+\delta\right)(t-s)} \, dt, \]

where \( \pi_{ijt} = (p_{ijt} - \bar{z}_{ijt}^{-}\eta)q_{ijt} - h_{ijt} - \lambda \) are profits of firm \( i \) producing variety \( j \). We solve this differential game focusing on Nash Equilibrium in open loop strategies. Let \( a_{ijt} = (q_{ijt}, h_{ijt}) \), \( t \geq s \), be a strategy for firm \( i \) producing \( j \) at time \( t \). Let us denote by \( a_{ij} \) firm \( i \) strategy path for quantities and innovation. At time \( s \) a vector of strategy path \( (a_{1j}, ..., a_{ij}, ..., a_{nj}) \) is an equilibrium in market \( j \) if

\[ V_{ijs}(a_{1j}, ..., a_{ij}, ..., a_{nj}) \geq V_{ijs}(a_{1j}, ..., a'_{ij}, ..., a_{nj}) \geq 0, \]

for all firms \( \{1, 2, ..., n\} \), where in \( (a_{1j}, ..., a'_{ij}, ..., a_{nj}) \) only firm \( i \) deviates from the equilibrium path. The first inequality states that firm \( i \) maximizes its value taking the strategy paths of the others as givens, and the second requires firm \( i \)'s value to be positive.\(^7\)

The characterization of the open loop Nash equilibrium proceeds as follows: a time \( s \) firm producing a particular variety solves (let us suppress indexes \( i \) and \( j \) to simplify notation)

\[ V_{s} = \max_{(q_{t}, h_{t})_{t=s}^{\infty}} \int_{s}^{\infty} \left[ (p_{t} - \bar{z}_{t}^{-}\eta)q_{t} - h_{t} - \lambda \right] e^{-\left(\rho+\delta\right)(t-s)} \, dt, \quad \text{st.} \quad (7) \]

\[ p_{t} = \frac{E}{X_{t}^{\alpha}} x_{t}^{\alpha-1} \]
\[ x_{t} = \hat{x}_{t} + q_{t} \]
\[ \dot{z}_{t} = A \, \bar{z}_{t} \, h_{t} \]
\[ \bar{z}_{s} > 0. \]

In a Cournot game a firm takes as given the path of its competitors’ production \( \hat{x}_{t} \), the path of the externality \( \bar{z}_{t} \), as well as the path of the aggregates \( E \) and \( X_{t} \), and the exogenous exit

\(^7\)We choose the open loop equilibrium because it is easier to derive in closed form solution. The drawback of focusing on the open loop equilibrium is that it does not generally have the property of subgame perfection, as firms choose their optimal time-paths strategies at the initial time and stick to them forever. In closed loop and in feedback strategies, instead, firms do not pre-commit to any path and their strategies at any time depend on the whole past history. The Nash equilibrium in this case is strongly time-consistent and therefore sub-game perfect. Unfortunately, closed loop or feedback equilibria generally do not allow a closed form solution and often they do not allow a solution at all. The literature on differential games has uncovered classes of games in which the open loop equilibrium degenerates into a closed loop and therefore is subgame perfect (e.g. Reingaum, 1982, Fershtman, 1987, and Cellini and Lambertini, 2005). A sufficient condition for the open loop Nash equilibrium to be subgame perfect is that in the first order conditions for a firm the state variable of other firms do not appear. In our model this condition is violated because of the externality in the innovation technology leading to the FOC (9) below. Although, none of the basic results of this paper depend on this externality, removing it complicates the solution of the model substantially.
rate \( \delta \). The first order conditions for the problem above are,

\[
\dot{z}_t^{-\eta} = \theta \frac{E}{X_t^\alpha} x_t^{\alpha-1},
\]

\[
1 = \frac{v_t A \dot{z}_t}{v_t},
\]

\[
-\eta \frac{z^{-\eta-1}}{v_t} q_t = -\frac{v_t}{v_t} + \rho + \delta,
\]

where \( v_t \) is the costate variable. From (8), firms charge a markup over marginal costs, with \( \theta \equiv (n - 1 + \alpha) / n \) being the inverse of the markup. This is the well known result in Cournot-type equilibria that the markup depends on the perceived demand elasticity, which is a function of both the demand elasticity and the number of competitors.

Firms producing the same variety are assumed to face the same initial conditions, resulting in a symmetric equilibrium with \( x_t = n q_t \). As shown in the appendix, substituting (8) into (1) we obtain the demand for variable inputs

\[
\dot{z}_t^{-\eta} q_t = \theta e \frac{z}{\bar{z}}
\]

where \( e \equiv E / n M \) is expenditure per firm, \( z \) is a measure of firm detrended productivity, \( z \equiv (\bar{z}_t e^{-gt}) \hat{\eta} \), with \( \hat{\eta} \equiv \eta \alpha / (1 - \alpha) \), and \( g \) is the growth rate of productivity that will be defined below. Average detrended productivity is

\[
\bar{z} \equiv \frac{1}{M} \int_0^M z_j \, dj.
\]

Notice that the amount of resources allocated to a firm in (11) is the product of average expenditures per firm, the inverse of the markup and the relative productivity of the variety the firm produces. When the environment becomes more competitive, \( \theta \) increases, prices lower, produced quantities increase and firms demand more inputs.

The right-hand side of equation (10) represents the return to innovation. After substituting \( v \) from (9), innovation returns become \( Ae (\bar{z}/\bar{z}) \dot{z}^{-\eta} q \). Since innovation is aimed at reducing production costs, \( z^{-\eta} q \), an increase in quantities makes innovation activities more profitable, inducing firms to innovative more.

Let us now define the externality \( \bar{z} \),

\[
\dot{z} = \frac{\bar{z}}{z} \bar{z},
\]

where by definition both the productivity \( \bar{z} \) and the detrended productivity \( z \) are assumed to represent the productivity of direct competitors, that is those producing the same variety. At the symmetric equilibrium they are both equal to the corresponding productivity of the firm.\(^8\)

Under this assumption, the growth rate of productivity

\[
g \equiv \frac{\dot{z}}{z} = \eta A \theta e - \rho - \delta,
\]

\(^8\)Notice that the externality \( \bar{z} \), can be rewritten as \( \dot{z} = z^{1-1/\theta} e^{gt} \bar{z} \). This implies that the first order condition for control \( h \), equation (9), depends on the state variable of direct competitors \( z \), except for the case \( \hat{\eta} = 1 \).
is the same for all $\tilde{z}$. To obtain it, differentiate (9) and substitute the resulting $\dot{v}/v$ in (10), then substitute $v\tilde{z}$ from (9) using the definition of $\tilde{z}$.

The particular assumption adopted for the externality $\tilde{z}$ allows for the growth rate to be equal across varieties, offsetting the positive effect that the relative productivity has on innovation and growth. Recall that more productive firms produce more and have then larger incentives to innovate. The externality has two components. First, there is a standard spillover effect coming from the productivity of direct competitors, as represented by $\tilde{z}$ in the definition of $\tilde{z}$. Second, there is a catching-up effect represented by the ratio $\tilde{z}/z$, introduced to offset the positive effect of the relative productivity on innovation. Equilibrium innovation for firm $z$ can be derived using (3), (12) and the definition of the externality,

$$h = \left[ \eta \theta e - \left( \frac{\rho + \delta}{A} \right) \right] \frac{z}{\tilde{z}},$$

where innovation resources $h$ are positively related to the firm’s relative productivity $z/\tilde{z}$. This is consistent with the empirical evidence showing that more productive firms spend more in innovation (e.g. Lentz and Mortensen, 2008, and Aw, Roberts, and Xu, 2010). The assumption of increasing innovation difficulty implies that, although more productive firms innovate more, all firm grow at the same rate in the steady-state equilibrium.\(^9\)

In a stationary equilibrium, all firms grow at the same rate and, as a consequence, their productivities grow at the same rate as the average productivity, meaning that their demand for variable inputs, as described by (11), is constant along the balance growth path. More importantly, the result that in a stationary equilibrium productivity grows at the same rate for all firms implies that firms remain always in their initial position in the productivity distribution.

### 2.4 Exit

From the previous section, it can be easily shown that the cash flow is a linear function of the relative productivity $z/\tilde{z}$

$$\pi(z/\tilde{z}) = (1 - \theta) e z/\tilde{z} - \left( \eta \theta e - \frac{\rho + \delta}{A} \right) \frac{z}{\tilde{z}} - \lambda.$$  

Produced quantities and innovation effort depend both on the distance from average productivity $z/\tilde{z}$. In the following, we assume $\eta$ small enough such that $1 - (1 + \eta)\theta > 0$, a sufficient condition for profits to depend positively on $z$. Let us denote by $z^*$ the stationary cutoff productivity below which varieties exit the market. At a stationary state, the cutoff productivity makes firm’s profits and firm’s value equal to zero, implying

$$e = \frac{\lambda}{z^*/\tilde{z}} - \frac{\rho + \delta}{A} \frac{1}{1 - (1 + \eta)\theta}.$$  

\(^9\)The assumption of increasing innovation difficulty is commonly used in R&D-driven growth models to eliminate counterfactual scale effects, and stationarize models with growing populations. Jones (1995) and Segertrom (1998) among others, provide robust empirical evidence supporting the increasing difficulty assumption.
We refer to it as the exit condition, a negative relation between \( e \) and \( z^* \).

Next, we assume that there is a mass of unit measure of potential varieties of which \( M \in [0, 1] \) are operative. We also assume that at each period non operative varieties draw a productivity \( z \) from an initial productivity distribution \( \Gamma(z) \), which is assumed to be continuous in \((z_{\text{min}}, \infty)\), with \( 0 \leq z_{\text{min}} < \infty \). Let us denote by \( \mu(z) \) the stationary equilibrium density distribution defined on the \( z \) domain. The endogenous exit process related to the cutoff point \( z^* \) implies \( \mu(z) = 0 \) for all \( z < z^* \). Since the equilibrium productivity growth rates are the same irrespective of \( z \), in a stationary environment, surviving firms remain always at their initial position in the distribution \( \Gamma \). Consequently, the stationary equilibrium distribution is \( \mu(z) = f(z)/(1 - \Gamma(z^*)) \), for \( z \geq z^* \), where \( f \) is the density associated to the entry distribution \( \Gamma \).

We can now write \( \tilde{z} \) as a function of \( z^* \)

\[
\tilde{z}(z^*) = \frac{1}{1 - \Gamma(z^*)} \int_{z^*}^{\infty} zf(z) \, dz. \tag{15}
\]

Since varieties exit at the rate \( \delta \), stationarity requires

\[
(1 - M)(1 - \Gamma(z^*)) = \delta M. \tag{16}
\]

This condition states that the exit flow, \( \delta M \), equals the entry flow defined by the number of entrants, \( 1 - M \), times the probability of surviving, \( 1 - \Gamma(z^*) \). Consequently, the mass of operative varieties is a function of the productivity cutoff \( z^* \),

\[
M(z^*) = \frac{1 - \Gamma(z^*)}{1 + \delta - \Gamma(z^*)}. \tag{OV}
\]

It is easy to see that \( M \) is decreasing in \( z^* \), going from \( 1/(1 + \delta) \) to zero.

Note that the entry distribution \( \Gamma \) is assumed to depend on detrended productivity \( z \). This assumption is crucial for the economy to be growing at a stationary equilibrium. Incumbent firms are involved in innovation activities making their productivity grow at the endogenous rate \( g \). This makes the distribution of incumbent firms move permanently to the right. By defining the entry distribution as a function of detrended productivity \( z \), we allow the productivity of entrants to grow on average at the same rate as that of incumbent firms. This is a form of technological spillover or learning-by-doing from incumbents to new entrants, sustaining a stationary equilibrium is a growing economy with exit and entry. A similar assumption has been previously used to support a stationary equilibrium in models of random (exogenous) growth with heterogeneous productivity such as Luttmer (2007), Poschke (2009) and Gabler and Licandro (2007).

\( \text{\textsuperscript{10}} \) Notice that problem (7) does not explicitly include positive cash flow as a restriction. By doing so and then imposing the exit condition (EC), we implicitly forbid firms with \( z < z^* \) to innovate and potentially grow at some growth rate smaller than \( g \). If they were allowed to do so, they will optimally invest in innovation up to the point in which the cash flow would be zero. In such a case, firms with initial productivity smaller than the cutoff value will be growing at a rate smaller than \( g \), moving to the left of the distribution and eventually exiting. Such an extension would make the problem unnecessarily cumbersome without affecting the main results.
2.5 Stationary Equilibrium

The market clearing condition for the homogeneous good can be written as

\[ n \int_0^M (y_j + h_j) \, dq_j + Y = n \int_0^M \left( z^{-\eta} q_j + h_j + \lambda \right) \, dq_j + \beta E = 1. \]

The total endowment of the homogeneous good is allocated to production of the composite good and to innovation, as well as to production of the homogeneous good. The first equality is obtained substituting \( y \) from (2) and \( Y \) from (4). Let us change the integration domain from sectors \( j \in [0,1] \) to productivities \( z \in [z^*, \infty] \) and use (3), (11) and (12) to rewrite the market clearing condition as

\[
\int_{z^*}^{\infty} \left( (1 + \eta) \theta e \, z/\bar{z} - \frac{\rho + \delta}{A} \frac{z}{\bar{z}} + \lambda \right) \, \mu(z) \, dz + \beta e = \frac{1}{nM}.
\]

Since \( \int_{z^*}^{\infty} \mu(z) \, dz = \int_{z^*}^{\infty} \frac{z}{\bar{z}} \mu(z) \, dz = 1 \), after integrating over all sectors we obtain

\[
e = \frac{1}{nM(z^*)} + \frac{\rho + \delta}{A} - \lambda, \quad \text{(MC)}
\]

a positive relation between \( e \) and \( z^* \).

The following assumption is needed to prove the existence of the stationary equilibrium.

**Assumption 1** The entry distribution verifies, for all \( z \),

\[ \frac{\bar{z}(z) - z}{\bar{z}(z)} \leq 1 - \frac{1}{zf(z)}, \tag{a} \]

and the following parameter restrictions hold:

\[ \lambda \frac{\bar{z}_e}{z_{\min}} > \frac{\rho + \delta}{A} \tag{b} \]

\[ 1 + \eta < \frac{A}{\theta} \tag{c} \]

where

\[ A = \frac{(1+\delta)\theta}{n} + \frac{\rho + \delta}{A} \left( 1 + \beta \right) - \lambda \left( 1 + \beta \frac{\bar{z}_e}{z_{\min}} \right) \]

\[ \frac{(1+\delta)\theta}{n} + \lambda \left( \frac{\bar{z}_e}{z_{\min}} - 1 \right) \tag{17} \]

and \( \bar{z}_e \) is the average productivity at entry.

Assumption (a) makes \( z^*/\bar{z}(z^*) \) increasing on \( z^* \), thus the (EC) curve is decreasing in \( z^* \).\(^{11}\)

Assumption (b) makes the profit function (14) increasing in \( z \). Assumption (c) guarantees that the (EC) curves cuts the (MC) curves from above.

**Proposition 1** Under Assumption 1, there exits a unique interior solution \((e, z^*)\) of (MC) and (EC), with \( M \) determined by (OV).

\(^{11}\)This assumption is similar to that needed in Melitz (2003) to make the exit condition (ZCP in the paper) decreasing in \( z^* \).
Proof. Since $M$ is decreasing in $z$, the (MC) locus is increasing starting at

$$
\frac{(1+\delta)}{n} + \frac{\rho + \delta}{A} - \lambda \\
\beta + (1 + \eta)\theta
$$

when $z^* = z_{\text{min}}$, and going to infinity when $z^*$ goes to infinity. Under Assumption 1(a), the (EC) locus is decreasing, starting at

$$
\frac{\lambda \frac{\bar{z}}{z_{\text{min}}} - \rho + \delta}{1 - (1 + \eta)\theta}
$$

for $z^* = z_{\text{min}}$, and going to $[\lambda - (\rho + \delta)/A] / [1 - (1 + \eta)\theta]$ when $z^*$ goes to $\infty$. Operating on the definition of $A$, it can be proved that assumption (b) implies $A < 1$, which from Assumption 1(c) implies $1 + \eta < 1/\theta$. Under this last condition, it can be proved that Assumption 1(c) is sufficient for the intercept of the (EC) curve be larger than the intercept of the (MC) curve, which completes the proof. Figure 1 provides a graphical representation of the equilibrium. ■

Next, we provide a first glance at the effects of trade openness by analyzing the effects of an exogenous increase in product market competition, a reduction in the markup rate $1/\theta$ which, as $\theta \equiv (n - 1 + \alpha)/n$, can potentially be produced by either an increase in the substitutability parameter $\alpha$, or by an increase in the number of firms $n$.

Proposition 2 An increase in $\theta$ raises the productivity cutoff $z^*$, reduces the number of operative varieties $M(z^*)$, has an ambiguous effect on the labor resources allocated to the homogeneous sector $e$ and increases the growth rate $g$.

Proof. Figure 1 shows the effect of an increase in the degree of competition (reduction in the markup $1/\theta$) on the equilibrium values of $z^*$ and $e$. An increase in $\theta$ shifts both the (EC) and the (MC) curves to the right, thereby increasing the equilibrium productivity cutoff $z^*$. Depending on the relative strengths of the shift of the two curves $e$ can increase or decrease, but the average growth rate $g$ always increases. From (12), the effect on $g$ of a change on $\theta$ is determined by its effect on $\theta e$. Multiplying the market clearing condition (MC) by $\theta$, we obtain $\theta e$ as a function of $\theta$ and $M(z^*)$, and since in equilibrium $M(z^*)$ is decreasing in $\theta$, we can conclude that $\theta e$ is increasing on $\theta$. ■

Two mechanisms contribute to increasing growth, a direct effect and a selection effect of competition. Let describe first the direct competition effect. In a Cournot equilibrium, an increase in competition reduces markups and allows for an increase in produced quantities; this can easily seen from (11) which shows that the quantity produced is positively related to $\theta$. The increase in quantities is feasible since the homogeneous good becomes relatively more expensive (i.e. the relative efficiency of the differentiate sector increases), consumers’ demand moves away from it towards the composite good and resources are reallocated from the homogeneous to the composite sector. Since the benefits of cost-reducing innovation are increasing in the quantity produced, the higher static efficiency associated with lower markups affects positively innovation.
and growth. This mechanism does not depend on firm heterogeneity: it is easy to check that assuming away the dependence of $M$ on $z^*$ by setting $M = 1$, the equilibrium growth rate derived from (MC) and (EC) becomes independent of the cutoff $z^*$, but still increasing on $\theta$. This direct effect of competition on growth can in fact be found in representative firm models of growth with endogenous market structure (see e.g. Peretto, 2003, and Licandro and Navas, 2007).

The selection effect is instead specifically related to the heterogeneous firms structure of the model. The trade-induced reduction in the markup raises the productivity threshold above which firms can profitably produce, the cutoff $z^*$, thus forcing the least productive firms to exit the market. As a consequence, market shares are reallocated from exiting to, more productive, surviving firms, thereby increasing their market size and their incentive to innovate. Therefore this selection effect leads to higher aggregate productivity level and higher innovation and productivity growth.\footnote{\textsuperscript{12}Notice that in this model the direct competition effect of trade liberalization on innovation does not hold if we eliminate the homogeneous good, because no reallocation of market shares would be possible. While the selection effect produced by the presence of firm heterogeneity would still hold because reallocation takes place within varieties of the differentiated product.}

### 3 Open economy

Consider a world economy populated by two symmetric countries with the same technologies, preferences, and endowments as described in the previous section. We assume that trade costs are of the iceberg type: $\tau > 1$ units of goods must be shipped abroad for each unit finally consumed. Costs $\tau$ can represent transportation costs or trade barriers created by policy. For simplicity in the baseline model we do not assume entry costs in the export market, thus all surviving firms sell both to the domestic and foreign markets.\footnote{\textsuperscript{13}Our main goal is to explain the interaction between trade, selection and innovation, and for this purpose having firms partitioned by their export status is not necessary.}

#### 3.1 Equilibrium characterization

Since the two countries are perfectly symmetric, we can focus on one of them. Let $q_t$ and $\tilde{q}_t$ be the quantities produced by a firm for the domestic and the foreign markets, respectively. Firms solve a problem similar to that in closed economy (see appendix). The first order conditions are:

\[
\begin{align*}
\tilde{z}_t^{\eta} &= \left(\alpha - 1 \right) \left( \frac{q_t}{x_t} + 1 \right) p_t \\
\tau \tilde{z}_t^{\eta} &= \left(\alpha - 1 \right) \left( \frac{\tilde{q}_t}{x_t} + 1 \right) p_t \\
1 &= v_t A \tilde{z}_t, \\
\frac{\eta \tilde{z}_t^{\eta-1}}{v_t} (q_t + \tau \tilde{q}_t) &= \frac{-\dot{v}_t}{v_t} + \rho + \delta.
\end{align*}
\]

Variable $x$ represents here the total output offered in the domestic market by both local and foreign firms. By symmetry it is equal to the total supply in the foreign market. Because of
the trade costs, firms face different marginal costs and set different markups for the domestic and foreign markets. More interesting, under Cournot competition countries export and import goods that are perfectly substitutable even at the cost of paying the variable trade cost.

In the appendix, we show that the first two conditions above yield the following demand for variable inputs

\[ \tilde{z}_t^\eta (q_t + \tau \tilde{q}_t) = \theta_r \tilde{z} \]

where \( \tilde{z} \) and \( \bar{z} \) are defined as in autarky and

\[ \theta_r = \frac{2n - 1 + \alpha}{n (1 + \tau)^2 (1 - \alpha)} \left[ \tau^2 (1 - n - \alpha) + n (2\tau - 1) + (1 - \alpha) \right] \]

is the inverse of the average markup faced by a firm in both the domestic and foreign market. Notice that \( \theta_r \) is decreasing in variable trade costs \( \tau \), with \( \theta_r \) reaching its maximum value \( \theta_{\text{max}} \equiv (2n - 1 + \alpha)/2n \) when \( \tau = 1 \), the polar case of no iceberg trade costs; the autarky value \( \theta = (n - 1 + \alpha)/n \) is reached when \( \tau = \bar{\tau} \equiv n/(n + \alpha - 1) \), the alternative polar case of prohibitive trade costs implying that both economies do not have any incentive to trade.

Using the last two first order conditions above and proceeding as in the closed economy, we find that the growth rate of productivity

\[ g_r \equiv \frac{\dot{z}}{\bar{z}} = \eta A \theta_r e - \rho - \delta \]

takes the same functional form as in the closed economy. Consequently, opening to trade only affects the equilibrium growth rate through changes in the markup. As in the closed economy case, we focus on the characterization of the steady-state equilibrium. The productivity cutoff is determined by the exit condition

\[ \pi(z^*/\bar{z}) = (1 - \theta_r) e z^*/\bar{z} - \left( \eta \theta_r e - \frac{\rho + \delta}{A} \right) \frac{z^*/\bar{z}}{\eta} - \lambda = 0. \]

which yields

\[ e = \frac{\lambda}{z^*/\bar{z} + \frac{\rho + \delta}{A}} - \frac{\rho + \delta}{A} \left( 1 - \frac{1}{\eta} \theta_r \right). \]

\[ (\text{EC}^T) \]

Since firms compensate their losses in local market shares with their shares in the foreign market, profits are only affected by the change in the markup. Consequently, the exit condition has the same functional form as in (EC) except for the \( \theta_r \).

The market clearing condition, proceeding as in the closed economy, becomes

\[ e = \frac{1}{nM(z^*)} + \frac{\rho + \delta}{A} - \frac{\lambda}{\beta + (1 + \eta) \theta_r}. \]

\[ (\text{MC}^T) \]

which is equal in all aspects to (MC) except for the markup, with \( \theta_r \) instead of \( \theta \). Equations (\text{EC}^T) and (\text{MC}^T) yield the equilibrium \((e, z^*)\) in the open economy, with \( M(z^*) \) determined by (OV). The equilibrium growth is defined by (20).
Proposition 3 Under Assumption 1 and for $\tau \in [1, \bar{\tau}]$, there exists a unique interior solution $(e, \bar{z}^*)$ of $(MC^T)-(EC^T)$.

Proof. At $\bar{\tau} = n/(n + \alpha - 1)$ the markups under trade and autarky are equal, $\theta_{\bar{\tau}} = \theta$, and the prohibitive level of trade costs is reached. Thus, for $\tau \geq \bar{\tau}$ firms do not have incentives to export, and trade does not take place. For $\tau < \bar{\tau}$ the proof of existence and unicity is similar to that in the closed economy, and we omit it for brevity.

3.2 Trade liberalization

When countries are symmetric, trade openness does not affect firms’ market shares because the reduction in local market sales due to foreign competition is offset by increased participation in the foreign market. For this reason, $(MC^T)$ and $(EC^T)$ are formally equivalent to $(MC)$ and $(EC)$ except for $\theta$. We can then apply proposition 2 to study the effects of trade liberalization. The economy with costly trade is characterized by a level of product market competition higher than in autarky, with $\theta_{\bar{\tau}} > \theta$, due to the participation of foreign firms in the domestic market. A larger number of firms in the domestic market raises product market competition, thus lowering the markup rate. From the definition of $\theta$ and the equilibrium value of $\theta_{\bar{\tau}}$ we obtain

$$\theta_{\bar{\tau}} - \theta = \frac{\tau (1 - \alpha)^2 - n (\tau - 1)^2 (n + \alpha - 1)}{n (1 + \tau)^2 (1 - \alpha)},$$

teta diff

which is positive for any non-prohibitive level of trade costs ($\tau < \bar{\tau}$). Differentiating the expression above it is easy to see that the distance between $\theta_{\bar{\tau}}$ and $\theta$ is decreasing in $\tau$, which implies that $\theta_{\bar{\tau}}$ is decreasing in $\tau$ (see appendix). Hence we have two results, first, when a country goes from autarky to costly trade, it experiences an increase in product market competition. Secondly, incremental trade liberalization increases product market competition as well. When trade is completely free, $\tau = 1$, product market competition reaches its maximum level, $\theta_{\text{max}} \equiv (2n - 1 + \alpha) / 2n$. Notice that $\theta_{\text{max}}$ has the same functional form as the inverse of the markup in autarky but with the number of firms doubled. Once established that trade reduces markups, it is easy to see that trade liberalization has the same effects on selection and innovation as those produced by an exogenous change in the markup in closed economy of proposition 2 shown in figure 1. These results can be summarized in the following proposition.

Proposition 4 Trade liberalization, both in the form of moving from autarchy to free trade and reducing variable trade costs, renders markets more competitive lowering markups, $1/\theta_{\bar{\tau}}$, and the number of operative varieties, $M$, and increasing the productivity cutoff, $z^*$, and the productivity growth rate, $g_{\bar{\tau}}$.

Similarly to the competition effect in closed economy, the trade-induced competition effect in open economy can be decomposed into two components: a direct effect induced by changes in the markup, which can be obtained also in a representative firm economy, and a selection effect produced by firm heterogeneity. The direct competition effect following trade liberalization is related to the reduction of oligopolistic inefficiency in the differentiated goods sector, which
raises the quantity produced by each firm.\textsuperscript{14} As innovation is cost reducing, the marginal benefit from a reduction in costs is increasing in the quantity produced, therefore lower markups trigger higher innovation. The \textit{selection effect} works through exiting of less productive firms triggered by the reduction in the markup: the market shares of exiting firms are reallocated towards surviving firms, thus increasing their quantity produced and their incentive to innovate. Thus, the selection effect of trade liberalization not only raises the level of productivity as in Melitz (2003) but also its growth rate.

Notice that trade liberalization has an anti-variety effect, it reduces the number of produced and consumed varieties $M$. This is a consequence of the assumption that there is a perfect overlap between the varieties produced by the two economies. The standard pro-variety effect of trade (e.g. Krugman 1980) could be generated by introducing asymmetry in the set of goods produced by the two countries. However, a model with asymmetric countries would complicate the algebra substantially, without adding much to the main mechanism we want to highlight (the effect of trade-induced selection on innovation and growth).

**Proposition 5** The growth effect of moving from autarky to costly or free trade is decreasing in $n$. While the growth effect of incremental trade liberalization is increasing in $n$.

As it can be easily seen from (19), the distance between open and closed economy markups is decreasing in $n$. This implies that opening up to trade is more beneficial, in terms of productivity gains, for less competitive countries. On the other hand, differentiating the absolute value of (19) with respect to $n$ we obtain

$$\frac{\partial (|\partial \theta_T/\partial \tau|)}{\partial n} = \frac{2(\tau - 1) (2n - 1 + \alpha)}{n^2 (1 + \tau)^3} > 0.$$  

Hence, once a country has opened to trade, further reductions in trade costs produce larger productivity gains the lower the oligopolistic inefficiency in the domestic market. Summarizing, less competitive closed economies benefit more from opening up to trade, and more competitive open economies experience a higher growth effect of further trade liberalization.

4 Discussion

The channel through which firms’ selection operates in this paper is different from the one in Melitz (2003). In Melitz, selection happens through the effects of trade on the labor market: trade liberalization increases labor demand, this bids up wages and the cost of production, thus forcing the least productive firms to exit the market. In our framework, the wage is constant, pinned down by the homogeneous good technology, and selection works through the effect of trade on product market competition: the reduction in the markup rate brought about by trade reduces profits and pushes the less productive firms out of the market. In Melitz this channel cannot operate because, under the assumption of monopolistic competition and CES preferences,

\textsuperscript{14}Recall that, since countries are symmetric, firms fully compensate the shares lost in the local market by an increase in their exports.
a larger number of competitors does not affect the elasticity of demand. In our oligopolistic model the market structure is endogenous and trade affects the distribution of surviving firms by raising competition in the product market. The two papers are complementary in that the wage channel of firms selection can be easily introduced in our model by removing the homogeneous good and let the wage be determined endogenously by the equilibrium in the labor market.

Another interesting difference with Melitz is that in his model firm heterogeneity does not play any role when the economy moves from autarky to free trade (zero trade costs): the effects of trade are exactly those found in the representative firm version of that model (Krugman, 1980). Firm selection takes place only with incremental trade liberalization (positive trade costs). In our model instead, the oligopolistic structure implies that firm selection takes place under radical trade liberalization as well. This happens because opening up to trade reduces markups even in the extreme case of free trade: the markup increases from its autarky level 1/\theta \equiv n/(n - 1 + \alpha) to the free trade level 1/\theta_r \equiv 2n/(2n - 1 + \alpha).

Finally, Melitz and Ottaviano (2008) features a selection effect of trade through the competition channel similar to ours, but the source of the endogenous market structure is different: they endogenize markups in a monopolistic competitive environment assuming an a non-homothetic structure of preferences that makes markups dependent on the number of firms (varieties). We instead, use a general preference structure and markups are pinned down by the strategic interaction between oligopolistic producers.

5 Extension: fixed export costs and endogenous $n$

We now explore the implications of extending the baseline model in two directions: endogenizing the number of firms, $n$, and allowing for an equilibrium with exporters and non-exporters. Following Melitz (2003), we assume that exporting firms face not only a variable trade cost but also a fixed export cost $\lambda_x$\footnote{As in Melitz, this is equivalent to a sunk cost for entering the export market: since productivity is known to the firm when they decide whether to export or not, firms are indifferent on whether to pay a sunk export cost $f_{ex}$ or its annualized value $\lambda_x \equiv f_{ex}/(\rho + \delta)$. Sunk export costs can be costs of setting distribution channels abroad, learning about foreign regulatory system, advertising etc.}. Since the focus is on the effect of trade on the productivity threshold, we keep matters simple by removing innovation. Under this assumption, the equilibrium distribution is $\mu(z) = f(z)/(1 - F(z^*))$, as in the benchmark model. Otherwise, the equilibrium distribution would be endogenous and the problem much harder to solve.

In a model of two symmetric countries, the fundamental difference between exporters and non-exporters is that the former face tougher competition. In fact, markets for non-exporters behave as in autarky but markets for exporters behave as under costly trade. The only difference between them is the markup they face, $1/\theta$ for non-exporters and $1/\theta_r$ for exporters, with $\theta$ and $\theta_r$ as defined above. Under these assumptions, the exit condition for non exporters is

\[
(1 - \theta)e = \frac{\lambda}{z} \left( \frac{1}{\rho \theta} \right)^{\frac{\alpha}{1 - \alpha}}, \tag{EC2}
\]
where
\[ \bar{p} = \left( \theta^{\frac{\alpha}{1-\alpha}} \int_{z^*}^{\infty} z \mu(z) dz + \theta_{e_{\tau}}^{\frac{\alpha}{1-\alpha}} \int_{z^*_e}^{\infty} z \mu(z) dz \right)^{\frac{\alpha-1}{\alpha}}, \]
is a geometric mean of varieties' prices. Notice that when \( \theta_{e_{\tau}} = \theta \), trade is too costly and the economy remains in autarky with the right-hand-side of (EC2) equal to \( \lambda z/\bar{z} \), as in the baseline model. There is a similar, new condition for firms participating in international trade
\[ (1 - \theta_{e_{\tau}}) e = \frac{\lambda + \lambda_x}{z^*_e} \left( \frac{1}{\bar{p} \theta_{e_{\tau}}} \right)^{\frac{\alpha}{1-\alpha}}, \quad (XC) \]
where \( z^*_e \) is the cutoff productivity for exporters. Variable production costs for non-exporters \( \bar{z} - q = e\theta^{\frac{1}{1-\alpha}} z \bar{p}^{\frac{\alpha}{1-\alpha}} \) and exporters \( \bar{z} - q_x = e\theta_{e_{\tau}}^{\frac{1}{1-\alpha}} z \bar{p}^{\frac{\alpha}{1-\alpha}} \) are derived in the appendix. Exit conditions (EC2) and (XC) differ in two elements: first, exporters pay both production fixed costs and export fixed costs. Second, the markup charged by exporters is smaller, since international markets are more competitive.

Notice that in our framework, highly productive firms facing international competition may make smaller profits than less productive local firms facing no international competition. All varieties are here potentially tradable, but some are not traded because of the fixed cost of export. Firms producing the non traded varieties are protected from international competition by the export fixed cost and benefit from larger markups. By combining (EC) and (XC), we get a linear relation between \( z^* \) and \( z^*_e \)
\[ \frac{z^*_e}{z^*} = \frac{1 - \theta}{1 - \theta_{e_{\tau}}} \left( \frac{\theta}{\theta_{e_{\tau}}} \right)^{\frac{\alpha}{1-\alpha}} \frac{\lambda + \lambda_x}{\lambda}. \quad (21) \]
Notice that the sign of \( d(z^*_e/z^*)/d\theta_{e_{\tau}} \) is positive, since it is equal to the sign of \( \theta_{e_{\tau}} - \alpha \), and \( \theta_{e_{\tau}} - \alpha > \theta - \alpha > 0 \). Moreover, it is also easy to see that \( z^*_e > z^* \) for any \( \tau \), since this is clearly true when \( \theta_{e_{\tau}} = \theta \) and then, given that \( d(z^*_e/z^*)/d\theta_{e_{\tau}} \) is positive, it has to be true for any other \( \theta_{e_{\tau}} > \theta \) as well.\(^{16}\)
Hence, in our oligopolistic framework no parameter restriction is needed to obtain the exporters non-exporters partition found in the data, while in the monopolistic competitive environment of Melitz (2003), the partition can only be obtained with sufficiently high export costs.

So far we have assumed that the number of firms producing a particular variety is exogenous. Here we extend the model to allow \( n \) to be pinned down by an entry condition. Firms entering the economy are assumed to pay a fixed entry cost \( \phi > 0 \) before they observe the productivity \( z \) of the good they will produce. Since profits are linear in productivity, free entry implies that the expected value of the firm must be equal to the entry cost
\[ (1 - \Gamma(z^*)) \frac{\bar{\pi}}{(\rho + \delta)} = \phi, \]
where the average profit is given by
\[ \bar{\pi} = \int_{z^*}^{z^*_e} \left[ (1 - \theta) e \theta^{\frac{\alpha}{1-\alpha}} z \bar{p}^{\frac{\alpha}{1-\alpha}} - \lambda \right] \mu(z) dz + \int_{z^*_e}^{\infty} \left[ (1 - \theta_{e_{\tau}}) e \theta_{e_{\tau}}^{\frac{\alpha}{1-\alpha}} z \bar{p}^{\frac{\alpha}{1-\alpha}} - \lambda - \lambda_x \right] \mu(z) dz \quad (22) \]
\(^{16}\)There is robust evidence that more productive firms self-select into the export market. See for instance, Bernard and Jensen (1999), Clerides, Lach, and Tybout (1998), and Aw, Chung, and Roberts (2000).
which yields the following expression for the free entry condition

\[(1 - \vec{\theta})e = \lambda + \left( \frac{1 - F(z^*)}{1 - F(z^*)} \right) \lambda_e + \left( \frac{\rho + \delta}{1 - F(z^*)} \right) \phi, \tag{FE} \]

where

\[\vec{\theta} = \theta \left( \frac{1}{\tilde{\theta}} \right)^{\frac{\alpha}{1 - \alpha}} \int_{z^*}^{z_\tau} z \mu(z) dz + \theta_\tau \left( \frac{1}{\tilde{\theta}_\tau} \right)^{\frac{\alpha}{1 - \alpha}} \int_{z^*}^{\infty} z \mu(z) dz,\]

is the average markup weighted by the varieties’ contribution to the average price.

Finally, in the market clearing condition we have to take into account that not all firms export, which leads to

\[\int_{z^*}^{\infty} \left[ e^{\theta_\tau^{\frac{1}{1 - \alpha}} z \tilde{\theta}_\tau^{\frac{\alpha}{1 - \alpha}} + \lambda} \right] \mu(z) dz + \int_{z^*}^{\infty} \left[ e^{\theta^{\frac{1}{1 - \alpha}} z \tilde{\theta}^{\frac{\alpha}{1 - \alpha}} + \lambda + \lambda_e} \right] \mu(z) dz + \beta e + \frac{1 - M(z^*)}{M(z^*)} \phi = \frac{1}{nM(z^*)}\]

where \((1 - M(z^*)) \phi / M(z^*)\) is the amount of resources devoted to entry. Using (16) and the definition of \(\vec{\theta}\) the market clearing condition can be written as

\[(\beta + \vec{\theta})e = \frac{1}{nM(z^*)} - \left[ \lambda + \left( \frac{1 - F(z^*)}{1 - F(z^*)} \right) \lambda_e + \frac{\delta}{1 - F(z^*)} \phi \right]. \tag{MC2} \]

A stationary equilibrium for this economy is a vector \(\{z^*, z^*_x, e, n\}\) solving the system (EC2)-(XC)-(FE)-(MC2), with \(M(z^*)\) determined by (OV). Since the equilibrium system is fairly complex, we explore its properties numerically in the quantitative section below.

6 Quantitative analysis

The purpose of this section is twofold: first, we explore the quantitative relevance of our mechanism, calibrating the baseline model and simulating trade liberalization scenarios. We calibrate the model’s steady state to match salient aggregate and firm level statistics of the US economy, then perform a counterfactual exercise analyzing the effects of a 10 percent reduction in the trade costs \(\tau\) on the innovation rate. Precisely, we quantitatively evaluate the effect of trade liberalization on innovation due to the direct competition effect, for which firm heterogeneity is not relevant, and to the selection effect, which pushes growth through a reallocation of market shares toward more productive firms. Secondly, we explore numerically the equilibrium properties of the extended version with endogenous \(n\) and fixed export costs, showing that the core results of proposition 2 hold in this more sophisticated economy as well.

6.1 Baseline calibration

Although the general analytical results presented above do not require assuming any particular productivity distribution, in order to perform our quantitative exercise we assume that the entry distribution is Pareto with shape parameter \(\kappa\), and scale \(z_{\text{min}}\). This is consistent with evidence on firm size distribution (e.g. Axtell, 2001, and Luttmer, 2007). In the baseline model, we have to calibrate 12 parameters \((\alpha, \tau, \delta, \beta, \lambda, n, L, \rho, \eta, A, \kappa, z_{\text{min}})\). The discount factor \(\rho\) is equal to the interest rate in steady state, thus we set it to 0.05 following the business cycle
literature. Anderson and Wincoop (2004) summarize the tariff and non-tariff barriers to trade using TRAINS (UNCTAD) data: for industrialized countries tariffs are on average 5% and non tariff barriers are on average 8%. We take the sum of these two costs and set \( \tau = 1.13 \). We set \( 1/\theta_* = 1.13 \) to match a 13 percent markup which is in the range of estimates in Basu (1994). We use these values of \( \tau \) and \( \theta \) and choose \( n = 6 \) such that equation (19) yields \( \alpha = 0.309 \) and therefore an elasticity of substitution across varieties of 1.44, which is in the range of existing macroeconomic estimates obtained in the international business-cycle literature (e.g. Heatcote and Perri, 2004, Ruhl, 2008, and Imbs and Mejean, 2009). Using Census 2004 data, we set \( \delta = 0.09 \) to match the average enterprise death rate in manufacturing observed in period 1998-2004.\(^{17}\) Rauch (1999) classifies goods into homogeneous and differentiated, and finds that differentiated goods represents between 64.6 and 67.1 percent of total US manufactures, depending on the chosen aggregation scheme. We use this result and calibrate the share of differentiated goods \( 1 - \beta = 0.66 \). We normalize the minimum value of the productivity distribution \( z_{\min} \) to 0.1; this normalization, as we show below, does not affect our quantitative results. Table 1 summarizes the calibration.

\[\text{[TABLE 1 ABOUT HERE]}\]

The remaining four parameters \((A, \eta, \lambda, \kappa)\) are calibrated internally in order to match some steady-state moments predicted by the model to key firm-level and aggregate statistics. Similarly to many calibrated models of firm dynamics we target the US economy, for which micro data are widely available (see i.e. Bernard et al., 2003, Luttmer, 2007, Alessandria-Choi, 2007). We use four targets, the first two are the average growth rate of productivity and the investment in innovation share of GDP. We use data from Corrado, Hulten and Sichel (2009) where US national account data have been revised to introduce investment in intangible capital, including R&D. Moreover, since there is no tangible capital in the model, all statistics used in the calibration must be adapted to the model economy. Precisely, the growth rate of labor productivity and the innovation to GDP ratio are obtained by subtracting investment in tangible capital from total income in the data. After this adjustment, Corrado et al. data report an average growth of labor productivity of 1.9% a year in the period 1973-2003. Since in the model all investment is in innovation, the targeted statistics for the innovation to GDP ratio is the investment in intangible capital share of total income; after subtracting tangible capital this leads to an average of 13.5% over the period 1973-2003. As shown in the appendix, where a more detailed description of the calibration strategy can be found, these two statistics are useful for calibrating the technological parameters \( A \) and \( \eta \).

We use two firm-level statistics to calibrate the fixed operating costs \( \lambda \) and the Pareto shape coefficient \( \kappa \). First, an average firm size of 21.8 workers found in Axtell (2001) for US firms in 1997 using Census data and considering only firms with at least one employee; we use this to calibrate \( \lambda \). Secondly, Bernard, Jensen, Eaton, and Kortum (2003) using 1992 Census data find a

\(^{17}\)For each year the death rates are computed as follows: taking year 2000 as an example, the death rate is the ratio of firms dead between March 2000 and March 2001 to the total number of firms in March 2000. Data can be downloaded at http://www.sba.gov/advo/research/data.html#ne, file data_uspdf.xls.
standard deviation of the productivity of US manufacturing firms of 0.75; we use this statistics to calibrate $\kappa$. Solving the systems of equation matching the data with the corresponding moments in the model we find: $A = 12.47$, $\eta = 0.0119$, $\lambda = 1.507$, and $\kappa = 2.621$.

6.2 Counterfactuals

In this exercise, we focus on quantifying the growth effect of a 10 percent reduction in the trade cost $\tau$, breaking it down into the direct competition effect and the selection effect highlighted in the model. For this purpose, let us define the direct competition effect by differentiating (20) but keeping the productivity threshold $z^*$ fixed, therefore ignoring the cutoff condition and using the market clearing condition ($MC^T$) to obtain the effect of $\tau$ on expenditure $e$. The resulting direct competition effect, denoted by $dg^d$, is

$$dg^d = \frac{\partial g_r}{\partial \theta_r} \frac{\partial \theta_r}{\partial \tau} + \frac{\partial g_r}{\partial e} \frac{\partial e}{\partial \theta_r} \frac{\partial \theta_r}{\partial \tau} = \beta (1 - \beta) \eta A e \frac{d\theta_r}{d\tau},$$

(23)

where $\partial e/\partial \theta_r$ was calculated using ($MC^T$) taking $z^*$ as given, and $\partial \theta_r/\partial \tau$ is calculated in the appendix. The selection effect is thus obtained as a residual $|dg^s_r| = |dg_r| - |dg^d_r|$, where $dg_r$ results from differentiating of $g_r$ with respect to $\tau$ in (20). We take the absolute values because trade liberalization implies a reduction in $\tau$.

[TABLE 2 ABOUT HERE]

Table 2 shows the effects of a 10 percent reduction in $\tau$ from its benchmark value of 1.13 to 1.117. It produces a 1.15 percent reduction in the markup. Both the productivity cutoff and innovation are fairly sensitive to changes in the markup. In fact the productivity cutoff $z^*$ rises by 4.21 percent, implying a 10.26 percent reduction in the survival probability of entering firms, $1 - F(z^*)$. The growth rate of aggregate productivity increases by 12.98 percent from 0.019 to 0.0214. Using (23) we find that only about 4 percent of the total increase in growth can be attributed to the direct effect, while the rest is produced by the selection effect. This suggests that the main mechanism highlighted in the paper, the innovation effect produced by the reallocation of market shares from exiting to surviving firms, is quantitatively relevant. In Table 2, we also show how doubling the benchmark value of parameters affects the results. As we can see, both the overall growth effect and the growth decomposition are very robust to parameters changes: the overall growth effect ranges between 0.129 and 0.233 percent increase in the growth rate, and about 92 to 97 percent of it can be attributed to selection.$^{18}$

In Table 3 below we compare our results with the findings of a representative sample of empirical and quantitative works performing a similar exercise. The scope of this comparison is twofold: first, it shows that existing empirical analyses have only studied some but not all the testable implications that our model can produce. Secondly, it shows that the quantitative predictions of our stylized model are fairly close to the existing empirical evidence.

$^{18}$We also performed the opposite exercise of halving the benchmark parameters obtaining similar results which we do not report for brevity.
Corcos, Del Gatto, Ottaviano and Mion (2007) estimate a version of the Melitz and Ottaviano (2008) model using firm level European data and find that a 5 percent reduction in trade costs reduces markups by 1.97 percent. Similarly, Chen, Imbs and Schott (2008) estimating the Melitz and Ottaviano (2008) model using European manufacturing firm-level data find that a 10 percent increase in the import to production ratio lowers the average markup by 1 percent. The elasticity of the markup to trade costs in our benchmark model is in the range of these two results. Aw, Roberts, and Xu (2010) using Taiwanese data estimate a dynamic structural model of firm’s decision to invest in R&D and to participate in the export market, with both activities affecting the dynamics of productivity. Their counterfactual exercise shows that a 5 percent reduction in the average tariff leads to a 5.3 percent increase in productivity in the long run (after 15 years).

Our findings are also in line with some recent reduced-form econometric analysis of trade-induced innovation and selection effects: Bloom, Draca, and Van Reenen (2009) for instance, find that a 10 percent increase in Chinese imports is associated with a 1.2 reduction in the probability of firm survival, a 21.4 percent increase in R&D. They also find that the selection effect, the “between component”, and the “direct effect” obtained controlling for labor reallocation, contribute equally to the increase in innovation resulting from increased trade with China. Additional evidence is provided by Bustos (2010): using Argentinean firm-level data she finds that a 24 percent reduction in Brazil’s tariff in the context of the MERCOSUR increased technology spending by Argentinean firms by an average 24 percent.\footnote{Technology spending includes several innovation activities such as R&D, computers, softwares, patents, and technology transfer.}

Summarizing, Table 3 provides two insights: first, it shows that several studies using different data and methodologies seem to suggest that the elasticity of the markup to a reduction in trade costs is the interval between 0.1 and 0.4, and that the elasticity of innovation to a reduction in trade costs roughly falls in the interval between 1 and 2. Secondly, it shows that none of the existing empirical works study the joint effect of trade on prices, firm survival, and innovation. Our dynamic general equilibrium model provides a specific mechanism linking trade-induced competition, firm selection, and innovation, and allows us to pin down the role of firm heterogeneity in shaping the effects of trade on productivity growth (growth decomposition). Hence, the paper lays down a new set of testable implications providing a theoretical guideline for future empirical work.

### 6.3 Extension

This section numerically solves the model with endogenous $n$ and fixed export costs. The main purpose of this exercise is to ask whether the basic results of the benchmark model hold in this richer environment. More precisely, we want to see if trade liberalization still increases product market competition and triggers firm selection when the number of firms is endogenous and markups and productivity thresholds are different for exporters and non-exporters. Since
the main focus here is qualitative, we do not recalibrate all parameters but take those used in
the benchmark model and calibrate only the two new parameters as follows: the fixed cost of
exporting $\lambda_x$ is set equal to 4.5 in order to match a productivity advantage of exporters of 33
percent, as found by Bernard et al. (2003) in Census 1992 US data. The sunk entry cost $\phi$ is
set to 0.00309 to generate an equilibrium $n = 6$ and consequently a 1.13 markup for exporting
firms, the values used in the benchmark calibration. We then perform the same counterfactual
exercise of reducing the trade cost by 10 percent. Figure 2 shows the results.

The three main results are: firstly, trade liberalization has a positive effect on $n$ and a
negative effect on the total number of firms $nM$. Second, both domestic and foreign markups
are reduced, thus trade liberalization increases competition for both exporters and non-exporters.
Third, tougher competition renders the domestic and the export markets more selective, thus
increasing both productivity cutoffs.

The key intuition behind these results is that the reduction in variable trade costs reduces
the exporters’ markup, forcing the less productive among them to exit the export market, thus
increasing the productivity cutoff $z^*_x$. The entry decision depends on expected profits which, as
can be seen in (22), are an average of domestic and export profits. From the baseline model
we know that $\theta < \theta_\tau$, therefore the profits of exporters are always lower than those of non
exporters. Since from (21) we know that the sign of $d(z^*_x/z^*)/d\theta_\tau$ is positive, a reduction
in $\tau$ by increasing $\theta_\tau$ and by reducing the share of exporters in the economy, increases the
average profits for entering firms $\bar{\pi}$ in (22) thus increasing entry and ultimately leading to a
higher equilibrium number of firms. Moreover, trade-induced increase in competition produces
a reallocation of resources from the homogeneous good to all varieties (exporters and non-
exporters) in the differentiated sector. This has an additional positive effect on the average
profits and induces more entry. A larger $n$ then reduces the domestic markup $1/\theta$ and raises the
domestic cutoff $z^*$ thereby forcing the least productive domestic firms to exit. Finally, a higher
$n$ also strengthens the reduction in the export markup produced by trade liberalization, thus
further increasing the export cutoff $z^*_x$.

The assumption that the two countries are perfectly symmetric, producing exactly the same
varieties, is key for interpreting these results. A reduction in variable trade costs makes exporting
more profitable, intensifying the presence of foreign firm in the domestic markets and viceversa.
This makes the export markets more competitive, reducing export markups and inducing the
marginal exporters to exit the foreign market. Notice that the effect of trade liberalization
on the export cutoff $z^*_x$ is different from that obtained in Melitz (2003). In that paper, as
in Krugman (1980), countries produce and trade different varieties, implying that there is no
direct competition between domestic and foreign goods. Therefore, reducing trade costs implies
that exporters benefit from an expansion of their market which leads to larger profits and
a lower productivity threshold for exporting. Our model can be extended to introduce the
standard pro-variety effect of trade by removing the assumption the two countries produce
exactly the same goods. As long as there is some overlap between domestic and foreign varieties, trade liberalization will reduce markups and make the domestic and the foreign markets more competitive (pro-competitive effect). The domestic cutoff will necessarily increase while the export cutoff could go up or down depending on how large is the overlap.

Another interesting result is that although lowering trade costs makes the export market more competitive, there are more exporters per variety \( n \) and each exporter trades more. In Figure 2, we can see that both the total number of domestic firms producing each variety, \( n \), and the average sales of exporters increase. These two predictions are in line with the empirical evidence on US firms.\(^{20}\) Interestingly, although trade liberalization increases the level of competition and reduces markups, there is an indirect ‘market-size’ effect that increases average firm size, sales and profits. Similarly to Melitz and Ottaviano (2008), the endogenous market structure of our model implies that trade liberalization has a positive effect on firms’ production that outweighs the direct competition effect on prices and markups and allows surviving firms to be bigger, sell more, and earn higher profits.

Table 4 shows the sensitivity of the results to changes in parameters values.

\[ \text{[TABLE 4 ABOUT HERE]} \]

As we can see, under all parameters specifications we obtain the same qualitative results. Quantitatively, it is worth noticing that the pro-competitive effect on both the domestic and the export market is lower with lower firm heterogeneity. Intuitively, a lower dispersion of firm productivity, higher \( \kappa \), reduces the role of trade-induced selection in both markets, a role that would completely disappear as firms become more homogeneous. Changing the rest of parameters does not seem to change the quantitative results significantly.

Finally, although we have simplified the model abstracting from innovation and growth, this simple comparative statics suggests that the economic mechanism behind the direct and selection effect of trade are still operative, and might actually be even stronger in the extended framework. Firstly, the reallocation of market shares from exiting to surviving firms is now accompanied by an additional reallocation from firms exiting the export market to surviving exporters. Secondly, the reallocation of market shares from the homogeneous good sector to all firms producing differentiated goods is still active and is reinforced by the stronger increase in competition brought about by the increase in the number of firms \( n \). With cost-reducing innovation these increases in the market share of surviving firms will boost their incentives to innovate, as in the benchmark model. Although it is fair to say that the qualitative results would hold in the more general framework, solving for the full model with an endogenous number of firms, sunk export costs, and innovation could definitely affect the quantitative results. This is certainly an interesting task for future research.

\(^{20}\) Bernard, Jensen and Schott (2006) find that a reduction in trade costs increases the volume of export. Bernard, Redding, and Schott (2010) find that the number of firms per product increases with a reduction in trade costs. Although they find that the number of both exporting firms and products increases, with the former increasing more than the latter. Below we discuss how our model can be extended to obtain the prediction that trade liberalization increases the number of products exported as well.
7 Conclusion

In this paper, we built a rich but tractable model of trade with heterogeneous firms and cost-reducing innovation, in order to account for a set of findings recently emerged from empirical works on the effects of trade liberalization. In our framework, the competition channel is at the roots of the selection and innovation effects of trade liberalization, as all other possible channels (pure market-size, international technology spillovers, terms of trade) have been excluded from the analysis. The response of market structure to trade liberalization derives directly from Cournot competition among firms. We have shown that trade liberalization reduces markups, thus forcing the less productive firms out of the market. This selection effect interacts with firms' innovation choice by redistributing resources towards the more productive firms and increasing their incentives to innovate, thereby increasing the aggregate long-run investment in innovation.

Calibrating the model to match US firm-level and aggregate statistics we show that the overall growth effect of a 10 percent reduction in variable trade costs is significant, and it is mainly attributable to the reallocation of resources across firms of different productivity levels triggered by firm selection. This suggests that firm heterogeneity can play a substantial role in analyzing the innovation and growth effects of trade liberalization.

The innovation effect of trade highlighted in our model suggests the existence of a new channel of welfare gains that has not been explored in the literature. To keep the model simple we have limited the analysis to the steady-state. A full understanding of the pro-competitive dynamic effects of trade requires the analysis of transitional dynamics, which we view as an interesting task for future research. Finally, studying two perfectly symmetric countries with an identical set of goods, does not allow us to obtain any pro-variety effects of trade. Introducing asymmetric countries is another important step for fully exploring the welfare effects of trade liberalization in our framework.

References


A Derivation of equation (11)

Rearranging (8), we obtain \( x = \tilde{z}^{\eta/(1-\alpha)} \frac{\theta E/X^\alpha}{\int_0^\infty \tilde{z}^{\eta} \, dz} \). Substituting it into (1) yields

\[
X^\alpha = \left( \frac{M}{\int_0^\infty \tilde{z}^{\eta} \, dz} \right)^{1-\alpha} (\theta E)^\alpha,
\]

where \( \tilde{\eta} \equiv \eta \alpha / (1 - \alpha) \). Using this into the expression for \( x \) above, we find

\[
x = \theta E \frac{\tilde{z}^{\eta/(1-\alpha)}}{\int_0^\infty \tilde{z}^{\eta} \, dz}.
\]

Substituting these results into (8) and considering that in a symmetric equilibrium \( x = nq \), we obtain

\[
\tilde{z}^{-\eta} q = \left( \frac{\theta E}{znM} \right)^{1-\alpha} q^\alpha = \theta e \tilde{z}.
\]

where \( e = E/(nM) \), \( z \equiv (\tilde{z}^t e^{-\eta t})^{\tilde{\eta}} \) and the average productivity

\[
\tilde{z} = \frac{1}{M} \int_0^M z_j \, dz_j.
\]

B Firm problem in open economy

Each firm solves the following problem

\[
V_s = \max_{(q_{D,t}^i, q_{F,t}^i, z_{D,t}, z_{F,t})} \int_s^\infty \left[ (p_{D,t} - \frac{1}{z_{D,t}}) q_{D,t}^D + (p_{F,t} - \frac{\tau}{z_{D,t}}) q_{F,t}^F - h_{D,t} - \lambda \right] e^{-\int_s^t (r_s + \delta) \, dz} \, dt
\]

\[
\text{s.t.}
\]

\[
p_{D,t} = \frac{E_{D,t}}{X_{D,t}^D} x_{D,t}^{\alpha-1} \quad \text{and} \quad p_{F,t} = \frac{E_{F,t}}{X_{F,t}^F} x_{F,t}^{\alpha-1}
\]

\[
x_{D,t} = \tilde{x}_{D,t}^D + q_{D,t}^D + x_{F,t}^D \quad \text{and} \quad x_{F,t} = \tilde{x}_{F,t}^F + q_{F,t}^F + x_{F,t}^F
\]

\[
\tilde{z}_{D,t} = A \tilde{z}_{D,t} h_{D,t}
\]

\[
z_{D,s} > 0,
\]

where \( p_{j,t}, E_{j,t} \) and \( X_{j,t} \) are the domestic price, expenditure and total composite good respectively for country \( j = D, F \), and \( q_i^t \) is the quantity sold from source country \( i \) to destination country \( j \). Writing down the current-value Hamiltonian and solving it yields the following first order conditions

\[
\left( \alpha - 1 \right) \frac{q_{D,t}^D}{x_{D,t}} + 1 \quad p_{D,t} = \frac{1}{z_{D,t}} \quad (24)
\]

\[
\left( \alpha - 1 \right) \frac{q_{F,t}^F}{x_{D,t}} + 1 \quad p_{F,t} = \frac{\tau}{z_{D,t}} \quad (25)
\]

\[
1 = v_{D,t} A \tilde{z}_{D,t} \quad (26)
\]

\[
\frac{\tilde{v}_{D,t} - \eta v_{D,t}^{-\eta-1}}{v_{D,t}} \left( q_{D,t}^D + \tau q_{F,t}^F \right) = -\tilde{v}_{D,t} + r_t + \delta \quad (27)
\]

30
Since the two countries are symmetric, \( q_{D,t}^{F} = q_{F,t}^{F} = q_{t} \), \( q_{D,t}^{F} = q_{F,t}^{F} = \dot{q}_{t} \), \( x_{D,t} = x_{F,t} = x_{t} \), \( E_{D,t} = E_{F,t} \), \( X_{D,t} = X_{F,t} \), \( p_{D,t} = p_{F,t} \). From (24) and (25) and using \( q_{t}/x_{t} + \dot{q}_{t}/x_{t} = 1/n \) yields

\[
\begin{align*}
\left[(\alpha - 1) \frac{q_{t}}{x_{t}} + 1\right] & = \frac{2n - 1 + \alpha}{n(1 + \tau)} = \theta_{D} \quad (28) \\
\left[(\alpha - 1) \frac{\dot{q}_{t}}{x_{t}} + 1\right] & = \frac{\tau 2n - 1 + \alpha}{n(1 + \tau)} = \theta_{F} = \tau \theta_{D} \quad (29)
\end{align*}
\]

which allows us to rewrite (24) and (25) as follows

\[
\theta_{D} \frac{E_{t}^{F}}{X_{t}^{F}} x_{t}^{\alpha - 1} = \frac{1}{z_{t}^{\eta}} \quad \text{and} \quad \tau \theta_{D} \frac{E_{t}^{F}}{X_{t}^{F}} x_{t}^{\alpha - 1} = \frac{\tau}{z_{t}^{\eta}}.
\]

Multiplying the above equations by \( q_{t} \) and \( \dot{q}_{t} \) and summing up we obtain

\[
\frac{q_{t} + \tau \dot{q}_{t}}{z_{t}^{\eta}} = n \left[ \theta_{D} \frac{q_{t}}{x_{t}} + \tau \theta_{D} \frac{\dot{q}_{t}}{x_{t}} \right] \frac{E_{t}}{X_{t}} \left( \frac{x_{t}}{X_{t}} \right)^{\alpha}.
\]

Using \( x_{t} = \{ [1/z_{t}^{\eta}] (X_{t}^{F}/\theta_{D} E_{t}) \}^{1-\alpha} \), it is easy to prove that \( (x_{t}/X_{t})^{\alpha} = \tilde{z}_{t} \). From (28) and using \( q_{t}/x_{t} + \dot{q}_{t}/x_{t} = 1/n \) we obtain

\[
\frac{q_{t} + \tau \dot{q}_{t}}{z_{t}^{\eta}} = \theta_{r} e_{t} \frac{z_{t}}{\tilde{z}_{t}} \quad (30)
\]

where

\[
\theta_{r} = \frac{2n - 1 + \alpha}{n(1 + \tau)^{2} (1 - \alpha)} \left[ \tau^{2} (1 - n - \alpha) + n (2 \tau - 1) + 1 - \alpha \right]
\]

is the inverse of the markup in the open economy.

### C Exit in open economy

The productivity cutoff is determined solving the following equation

\[
\pi_{t}(\tilde{z}^{*}) = \left( p_{t} - \frac{1}{\tilde{z}_{t}^{\eta}} \right) q_{t} + \left( p_{t} - \frac{\tau}{\tilde{z}_{t}^{\eta}} \right) \dot{q}_{t} - h_{t} - \lambda = 0
\]

Using \( p_{t} = 1/\theta_{D} \tilde{z}_{D,t}^{\eta} \) and \( h_{t} = \eta \theta_{r} e_{t} \tilde{z}_{t} - (\rho + \delta)/A \) obtained from (26) and (27) yields

\[
\frac{1}{\theta_{D}} \frac{q_{t} + \dot{q}_{t}}{\tilde{z}_{t}^{\eta}} - \left( \frac{q_{t} + \tau \dot{q}_{t}}{\tilde{z}_{t}^{\eta}} \right) (1 + \eta) + \frac{\rho + \delta}{A} - \lambda = 0.
\]

With the same procedure used to derive (30) we obtain

\[
\frac{q_{t} + \dot{q}_{t}}{\tilde{z}_{t}^{\eta}} = \theta_{D} e_{t} \frac{z_{t}}{\tilde{z}_{t}}
\]

which, together with (30), yields

\[
[1 - (1 + \eta) \theta_{r}] e_{t} z_{t}^{*}/\tilde{z}_{t} + \frac{\rho + \delta}{A} - \lambda = 0.
\]

This expression is similar to (EC) except for the markup \( 1/\theta_{r} \) in the place of \( 1/\theta \).
D Pro-competitive effect

Differentiating $\theta_\tau$ with respect to $\tau$

$$\frac{\partial \theta_\tau}{\partial \tau} = -\frac{2(\tau - 1)(2n - 1 + \alpha)^2}{n(1 + \tau)^3(1 - \alpha)} \leq 0,$$

thus trade liberalization reduces the markup. Moreover, taking the absolute value of this derivative and differentiating it with respect to $n$ we find

$$\frac{\partial (|\theta_\tau| / \partial \tau)}{\partial n} = \frac{2(\tau - 1)(2n - 1 + \alpha)}{n^2(1 + \tau)^3} > 0,$$

which implies that the competition effect of incremental trade liberalization is decreasing in the number of firms $n$.

E Calibration

Let us denote the vector of externally calibrated parameters with $\Omega$ and solve the equilibrium system (EC)-(MC) to obtain $e$ and $z$ as functions of $\Omega$ and the four parameters that we have to calibrate, $A$, $\eta$, $\lambda$, $\kappa$. Let us call $\Theta = (A, \eta, \lambda, \kappa, z_{\min})$ the vector of parameters calibrated internally. We then use the moments in the model corresponding to the statistics we want to match. The first moment is the average growth rate of production

$$g_q = \eta g (1 - \beta) = \eta [\eta A \theta_\tau e (\Omega, \Theta) - \rho - \delta] (1 - \beta)$$

obtained from (20) and using the production function (2) and the fact that a share $\beta$ of the economy does not innovate. Similarly the innovation share of income in our model is

$$r = \frac{\eta \theta_\tau e (\Omega, \Theta) - \rho + \delta}{\epsilon (\Omega, \Theta)(1 + \beta)nM}$$

where from (13) we know that the resources devoted to innovation by firm $z$ are $h(z) = gz/A\bar{\varepsilon}$, thus average innovation is $h = \eta \theta_\tau e (\Omega, \Theta) z - (\rho + \delta)/A$. These two moments are relevant for the calibration of $A$ and $\eta$, since these are technological parameters affecting the return to innovation.

From (2) we obtain the average firm size

$$\bar{y} = \theta_\tau e (\Omega, \Theta) + \lambda$$

which is relevant in calibrating the fixed cost $\lambda$. Finally, a relevant moment in calibrating the shape parameter of the Pareto distribution of firm productivity $\kappa$, is the standard deviation of firm productivity

$$std_z = \frac{z^*(\Omega, \Theta)\kappa^{\frac{1}{2}}}{(\kappa - 1)(\kappa - 2)^{\frac{1}{2}}}$$

where we can use (EC) and (MC) to express $z^*(\Omega, \Theta)$ as a function of parameters. Using the statistics discussed in the text, namely $g_q = 0.019$, $r = 13.5$, $\bar{y} = 21.8$ and $std_z = 0.75$, this system of equations is used to calibrate the vector of parameters $\Theta = (A, \eta, \lambda, \kappa)$ obtaining the results shown in the main text.
F Equilibrium quantity for exporters and non-exporters

We want to derive the variable costs (or productivity-adjusted quantities) for non-exporters $\tilde{z}^{-\eta}q$ and exporters $\tilde{z}^{-\eta}q_x$. Proceeding as in the benchmark model, from (8) we obtain $x^\alpha = \tilde{z}^{-\eta}(\theta E/X^\alpha)^{1/\alpha}$ and substituting into (1) yields $(x/X)^\alpha = \theta^{\alpha/\alpha}z/M\tilde{p}$. Substituting this back into (8) we obtain

$$\tilde{z}^{-\eta} q = \theta^{1/\alpha} ez\tilde{p}^{\alpha/\alpha},$$

where $e = E/nM$ and $z$ is a measure of detrended productivity, $ze^{\eta t} = z_t^{\eta}$. With the same procedure we obtain the productivity-adjusted quantity for exporters

$$\tilde{z}^{-\eta} q_x = \theta^{1/\alpha} ez\tilde{p}^{\alpha/\alpha}.$$

Using these two results we can easily determine the domestic cutoff condition (EC2) and the export cutoff condition (XC).
## Table 1
### Summary of Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Moment</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.309</td>
<td>Elasticity of sub/markup</td>
<td>Ruhl (2008)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.09</td>
<td>Enterprise death rate</td>
<td>US Census (2004)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.34</td>
<td>Share non differentiated</td>
<td>Rauch (1999)</td>
</tr>
<tr>
<td>$n$</td>
<td>6</td>
<td>Elasticity of sub/markup</td>
<td>Basu (1994)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.05</td>
<td>interest rate</td>
<td>Mehra-Prescott (2005)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.0119</td>
<td>Innovation/GDP, Growth</td>
<td>CHS (2009)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.507</td>
<td>avg. firm size</td>
<td>Axtell (2001)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>2.621</td>
<td>std. firm productivity</td>
<td>BJEK (2003)</td>
</tr>
</tbody>
</table>

## Table 2
### Sensitivity Analysis

(doubling the benchmark)

| Source | $\frac{1}{\theta_{\tau}}$ | $z^*$ | $1 - F(z^*)$ | $\tilde{y}$ | $|dg_{\tau}|$ | $|dg_{\tau}^d|$ | $|dg_{\tau}^s|$ |
|--------|-----------------|------|---------------|-------------|-------------|-----------------|-----------------|
| bench  | markup          | cutoff | survival      | size        | growth      | direct          | selection       |
| $n = 12$ | $\kappa = 5.24$ | $\lambda = 3$ | $\beta = 0.68$ | $\delta = 0.18$ | $\eta = 0.0218$ | $|dg_{\tau}| = 0.1965$ | $|dg_{\tau}^d| = 4.2\%$ | $|dg_{\tau}^s| = 95.8\%$ |

**Benchmark:** $n = 6, \kappa = 2.62, \lambda = 1.5017, \beta = 0.34, \delta = 0.09, \eta = 0.0109$
Table 3

COMPARISON WITH EMPIRICAL EVIDENCE

<table>
<thead>
<tr>
<th>Moments model</th>
<th>CIS</th>
<th>CDMO</th>
<th>BDV</th>
<th>ARX</th>
<th>BUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\theta_{\tau}$</td>
<td>-0.0115</td>
<td>-0.01</td>
<td>-0.019</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1 - F(z^*)$</td>
<td>-0.1026</td>
<td></td>
<td>-0.012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>d_{g_{\tau}}</td>
<td>$</td>
<td>0.1298</td>
<td>0.24</td>
<td>0.053</td>
</tr>
<tr>
<td>$</td>
<td>d_{g_{\tau}^D}</td>
<td>$</td>
<td>4.2%</td>
<td>50%</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>d_{g_{\tau}^X}</td>
<td>$</td>
<td>95.8%</td>
<td>50%</td>
<td></td>
</tr>
</tbody>
</table>

Sources: Chen et al. (2008), Corcos et al. (2007), Bloom et al. (2009), Aw et al. (2010), Bustos (2010)

Table 4

SENSITIVITY ANALYSIS: EXTENDED MODEL
(doubling the benchmark)

<table>
<thead>
<tr>
<th></th>
<th>bench</th>
<th>$\lambda = 3$</th>
<th>$\beta = 0.68$</th>
<th>$\delta = 0.18$</th>
<th>$\phi = 0.0061$</th>
<th>$\kappa = 5.24$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\theta$</td>
<td>markup D</td>
<td>-0.0004</td>
<td>-0.0003</td>
<td>-0.0005</td>
<td>-0.0002</td>
<td>-0.0005</td>
</tr>
<tr>
<td>$1/\theta_{\tau}$</td>
<td>markup EX</td>
<td>-0.0116</td>
<td>-0.0109</td>
<td>-0.0103</td>
<td>-0.0128</td>
<td>-0.0082</td>
</tr>
<tr>
<td>$z^*$</td>
<td>cutoff D</td>
<td>0.0026</td>
<td>0.0023</td>
<td>0.00288</td>
<td>0.0017</td>
<td>0.0023</td>
</tr>
<tr>
<td>$z_{\tau}$</td>
<td>cutoff EX</td>
<td>0.0931</td>
<td>0.0874</td>
<td>0.0817</td>
<td>0.1034</td>
<td>0.0609</td>
</tr>
<tr>
<td>$n$</td>
<td>firms</td>
<td>0.0032</td>
<td>0.0027</td>
<td>0.0036</td>
<td>0.0019</td>
<td>0.0030</td>
</tr>
<tr>
<td>$nM$</td>
<td>total firms</td>
<td>-0.0068</td>
<td>-0.0062</td>
<td>-0.0074</td>
<td>-0.0045</td>
<td>-0.0061</td>
</tr>
<tr>
<td>$s_x$</td>
<td>avg. sales EX</td>
<td>0.1272</td>
<td>0.1567</td>
<td>0.1002</td>
<td>0.1501</td>
<td>0.0640</td>
</tr>
</tbody>
</table>

Benchmark: $\kappa = 2.62, \lambda = 1.5017, \beta = 0.34, \delta = 0.09, \phi = 0.00309$
Figure 1. Steady-state equilibrium

\[ \frac{\lambda \frac{z_c}{z_{\text{min}}} - \rho + \delta}{1 - (1 + \eta)\theta} \]

\[ \frac{(1 + \delta) L - \rho + \delta}{\frac{n}{\beta + (1 + \eta)\theta} - \lambda} \]
Figure 2: Trade liberalization with endogenous n and fixed export costs