

# Fiscal policy, composition of intergenerational transfers, and income distribution\*

Jaime Alonso-Carrera

Departamento de Fundamentos del Análisis Económico and RGEA

Universidade de Vigo

Jordi Caballé

Institute for Economic Analysis (CSIC) and  
UFAE and CODE (Universitat Autònoma de Barcelona)

Xavier Raurich

Departament de Teoria Econòmica and CREB

Universitat de Barcelona

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## Abstract

In this paper, we characterize the relationship between the initial distribution of human capital and physical inheritances among individuals and the long-run distribution of these two variables. In a model with borrowing constraints and indivisible investment in education, we discuss how the initial composition of intergenerational transfers determines the posterior intergenerational mobility in human capital and the evolution of intragenerational income inequality. This analysis enables us in turn to characterize the effects of fiscal policy on future income distribution, intergenerational mobility, and economic performance when the composition of intergenerational transfers is endogenous.

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## 1. Introduction

The question of how inequality is generated and how it evolves over time is one of the major concerns in economic analysis. In the last decades a large number of studies have provided evidence supporting the presumption that intergenerational transfers are key to explain the empirical distributions of income and wealth.<sup>1</sup> Intergenerational transfers may take the form of physical capital (bequests) or human capital (investment in education). Empirical evidence also shows that both types of transfers affect income distribution. In this paper, we follow this line of research and show that the initial distribution of bequests and human capital, as well as fiscal policy, determines the stationary composition of intergenerational transfers that individuals leave to their offspring.

Investment in education is a key factor of income inequality.<sup>2</sup> As was pointed out by Galor and Zeira (1993), there are two main features that give rise to this relationship. On the one hand, the technology of human capital accumulation exhibits a non-convexity since the investment in education is indivisible. This technological feature imposes some liquidity constraints on the poorest individuals so that their access to education depends on whether they can borrow or not. On the other hand, there are capital market imperfections resulting in borrowing constraints so that those individuals with an income level below some threshold level can not afford the cost of education.<sup>3</sup> Therefore, the initial distribution of income determines the number of individuals who can acquire education and, thus, it determines the aggregate stock of human capital and the rate of economic growth. This mechanism linking education with income distribution and growth was already widely analyzed in the literature by authors like, for instance, Galor and Zeira (1993), García-Peñalosa (1995), Galor and Tsiddon (1997), and Owen and Weil (1998), among others.

The previous relationship between education and income distribution implies that intergenerational transfers from parents to children account for a part of the observed inequality. Obviously, intergenerational transfers help to reduce the negative effects of liquidity and borrowing constraints on the accumulation of human capital. Note in this respect that intergenerational transfers can take two forms: (i) transfers of physical capital by means of bequests; and, (ii) transfers of human capital by means of the parents' investment in the education of their children. When parents do not pay the education cost and, thus, only leave bequests to their offspring, only those individuals who receive a sufficiently large inheritance and, thus, do not need to borrow can acquire human capital. Galor and Zeira (1993) show that, if one assumes credit market imperfections and a non-convex education technology, then the inherited distribution of wealth affects the accumulation of human capital and the dynamics of income distribution. When education is financed by parents, only those individuals whose parents have a sufficiently high level of income have access to education (see Becker and Tomes,

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<sup>1</sup>See, for instance, Becker and Tomes (1986), Gokhale et al. (1999), Gokhale and Kotlikoff (2002), Laitner (2002) or Wolf (2002), among many others.

<sup>2</sup>García-Peñalosa (1994) or Aghion et al. (1999) review the literature that examines the role of education on the link between distribution and growth.

<sup>3</sup>See, for instance, Dynarski (2002) or Keane (2002) for a discussion of the role of borrowing constraints on decisions concerning human capital acquisition.

1976; Eckstein and Zilcha, 1994; or Behrman et al., 1995).

The literature that we have reviewed above has not considered simultaneously the two types of intergenerational transfers we have mentioned: physical bequests and education. Then, one could ask whether the coexistence of these two types of transfers affects the intragenerational income distribution and the intergenerational earnings mobility. In this paper, we address this question by considering the interaction between the composition of intergenerational transfers and income distribution. We show that the initial mix of these two types of transfers is a key variable to understand the relationship between investment in education and income distribution. In a related paper, Zilcha (2003) also arrives at the conclusion that the composition of these transfers may end up determining the intragenerational distribution of income and the pattern of capital accumulation through a model where the interaction between those variables is based on an "ad-hoc" mechanism. Since this author intends to show that differences in the composition of intergenerational transfers may explain at least part of the observed differences in growth and inequality across countries, he assumes that this composition is exogenously given. In particular, he assumes a "joy of giving" motive for intergenerational transfers where parents' marginal utilities with respect to bequest and transfers of human capital are different. However, this way of modelling the link between the composition of intergenerational transfers and income distribution imposes a rigid constraint on the analysis of the determinants of both the intragenerational income distribution and the intergenerational mobility in human capital. In contrast, we consider that the composition of intergenerational transfers is endogenously determined by other economic factors like education costs, borrowing constraints or fiscal policies without introducing any differential treatment at the preference level between these two types of transfers.

Our paper develops a model of a small open economy populated by overlapping generations of individuals who differ in the level and the composition of inherited transfers from parents. In this economy the disposable lifetime income of an individual is fully determined by the bequest and human capital inherited from his parent. These intergenerational transfers arise because individuals care about the starting opportunities of their children so that parents take into account the disposable income of their offspring. More precisely, we assume that parents derive utility from their contribution to the future lifetime income of their children without discriminating between the types of intergenerational transfers used for making such a contribution.<sup>4</sup> Thus, in our model there is an endogenous trade-off between the two types of intergenerational transfers, which is driven by their relative returns. Finally, since we assume that the investment in education is indivisible and that individuals can not borrow to finance the education of their children, those individuals with an income level below some threshold do not finance the cost of education of their children and, thus, they only leave bequest to their offspring. In this way, both the initial distribution and the composition of wealth drive the evolution of the composition of intergenerational transfers and, thus, they determine the size of the educated population along the equilibrium path. This simple

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<sup>4</sup>Becker and Tomes (1986) defend this formulation of altruism. Our notion of altruism lies thus between the "joy-of-giving" motive, where individuals receive direct utility from the act of giving, and "family altruism", where individuals' felicity depends on the disposable income of their children. See Michel et al. (2006) for a comparison between different forms of altruism.

mechanism explains how the initial distribution of wealth determines the evolution of intragenerational income inequality and of intergenerational mobility in human capital. As the income of an individual depends on the value of his inheritance, we will see that an individual finances the education of his children if he has received an inheritance that is larger than some threshold level. This threshold level of bequest for educated individuals differs from the threshold level for non-educated individuals since educated individuals earn a higher labor income as a result of the education premium. Therefore, the access to education of individuals does not depend only on the transfers received from their parents, but also on the transfers that their parents have received.

A natural question to ask in our model is how different fiscal policies affect the evolution of both the intragenerational income distribution and the intergenerational mobility in human capital. In this paper we analyze the effects of the following government interventions: a pay-as-you-go social security system, a tax on inheritance, a tax on capital income, a tax on labor income, and a subsidy on education investment. These fiscal instruments affect the composition of intergenerational transfers, which in turn modifies income distribution, intergenerational mobility, and economic performance. In particular, we obtain that both the inheritance and the capital income taxes increase the fraction of non-educated individuals in the total population and reduce the level of bequest per capita, whereas the labor income tax also reduces the level of bequests but may raise the fraction of educated individuals if the education premium is sufficiently large. The effects of the social security system depends on whether the economy is dynamically efficient or inefficient as defined by Cass (1972). If the economy is dynamically inefficient (efficient), the effects of social security are qualitatively identical (the opposite) to those arising from inheritance taxation. Finally, we show that the stationary fraction of educated individuals under a partially public system of education is larger than under a private system of education for the same initial distribution of bequests and human capital.

The paper is organized as follows. Section 2 presents the model of overlapping generations with altruistic individuals. Section 3 solves the intertemporal choice problem faced by an individual. In section 4 we describe the dynamics of the distribution of bequests and human capital following a given initial distribution. Section 5 analyzes the effects of fiscal policy on the intergenerational mobility in human capital and on the stationary distribution of income. Section 6 concludes the paper.

## 2. The model

We consider a small open economy populated by overlapping generations of individuals who live for three periods. There is a continuum of dynasties distributed on the interval  $[0, 1]$ . A new generation of individuals is born in each period within each dynasty. Each individual has offspring at the beginning of the second period of his life and the number of children per parent is  $n \geq 1$ . An agent makes economic decisions only during the last two periods of his life. In every period, the youngest individuals neither consume nor work, but they can accumulate human capital by attending formal school. Individuals work and supply inelastically one unit of labor when they are adult (second period of life) and are retired when they are old (third period of life). Individuals are assumed to care about the future income of their children and they can give two kinds of transfers to

them: physical bequests and education. We will use the convention that the generation  $t$  is composed of the individuals who are adult (workers) in period  $t$ . As we will see next, all the individuals belonging to the same dynasty  $i \in [0, 1]$  and to the same generation  $t$  are identical in all respects.

In this economy there is a government that collects revenues from taxing proportionally labor income at the rate  $\tau_w \in [0, 1]$ , capital income at the rate  $\tau_k \in [0, 1]$ , and inheritances at the rate  $\tau_b \in [0, 1]$ , and from lump-sum taxes. The government spends those revenues to finance a subsidy to education at the rate  $s_e \in [0, 1]$ , and public consumption. Finally, the government faces a balanced budget constraint in each period. Thus, the government is subject to the following budget constraint at period  $t$ :

$$n \cdot \left[ \int_{[0,1]} (\tau_w w_t h_t^i + \tau_b b_t^i - s_e n e_t^i + \lambda) di \right] + \int_{[0,1]} (\tau_k r_t s_{t-1}^i + \theta) di = G_t, \quad (2.1)$$

where  $G_t$  denotes average government consumption per old individual at period  $t$ ;  $\lambda$  and  $\theta$  are the lump-sum taxes faced by individuals at adult and old periods, respectively;  $w_t$  is the wage per efficiency unit of labor at period  $t$ ;  $r_t$  is the before-tax net rate of return on saving at period  $t$ ;  $b_t^i$  is the amount of bequests at period  $t$  that an old individual of dynasty  $i$  (who is born in period  $t-2$ ) leaves to each of their direct descendants (who were born in period  $t-1$ );  $h_t^i$  is the level of human capital of an adult individual belonging to dynasty  $i$  and generation  $t$ ;  $s_{t-1}^i$  is the amount saved at the end of adulthood by an individual of dynasty  $i$  and generation  $t-1$ , and  $e_t^i$  denotes the income that the adult individual of dynasty  $i$  and generation  $t$  devotes to finance the education of each of their children. The number of efficiency units of labor supplied by an individual belonging to dynasty  $i$  and generation  $t$  is equal to his level  $h_t^i$  of human capital. We assume that public consumption neither affects directly individuals' welfare nor participates in the production process.

Individuals derive utility from both their own lifetime consumption and their contribution to the lifetime income of their children. Preferences of an individual belonging to dynasty  $i$  and generation  $t$  are represented by the utility function:

$$U_t^i = \ln c_t^i + \rho \ln x_{t+1}^i + \beta \ln I_{t+1}^i, \quad (2.2)$$

where  $\rho > 0$  is the temporal discount factor, the coefficient  $\beta > 0$  measures the intensity of altruism,  $c_t^i$  and  $x_{t+1}^i$  are the amount of consumption in the second and third periods of life, respectively, and  $I_{t+1}^i$  is the after-tax contribution to the future lifetime income of each of their children. We assume that individuals do not discriminate among their children so that they make the same contribution  $I_{t+1}^i$  for all their direct descendants<sup>5</sup>. The inheritance  $b_t^i$  and the level of human capital  $h_t^i$ , together with the interest rate, wages per efficiency unit, and fiscal policy, determine the value of the after-tax lifetime income of an individual belonging to dynasty  $i$  and generation  $t$ . The parental contribution to the income of this individual is then given by

$$I_t^i = (1 - \tau_w) w_t \Delta_t^i + (1 - \tau_b) b_t^i, \quad (2.3)$$

<sup>5</sup>The altruism parameter  $\beta$  can thus be rewritten as  $\beta = n\rho\beta'$ , where  $\beta'$  would denote the pure altruism factor per descendant.

where  $\Delta_t^i$  is the increase in the stock of human capital that this individual has acquired.

We assume that the human capital of an individual is entirely determined by his parent's investment in education. In particular, the human capital level of an individual can take two values depending on whether his parent investment in his education is below or above the fixed cost of education  $\mu$ . Thus, the level of human capital at period  $t+1$  of an adult individual belonging to dynasty  $i$  (who is born at period  $t$ ) is given by the following equation:

$$h_{t+1}^i = 1 + \Delta_{t+1}^i, \quad (2.4)$$

with

$$\Delta_{t+1}^i = \begin{cases} 0 & \text{if } e_t^i < \mu \\ \varepsilon & \text{if } e_t^i \geq \mu \end{cases},$$

where  $\varepsilon > 0$  and  $\mu > 0$ . Obviously, the optimal investment in education for the individuals who want to have uneducated children (with  $h_{t+1}^i = 1$ ) is  $e_t^i = 0$ , whereas those individuals who want educated children (with  $h_{t+1}^i = 1 + \varepsilon$ ) will choose  $e_t^i = \mu$ .

There is a single commodity that can be devoted to either consumption or investment, and the investment can be either in physical or in human capital. Adult individuals distribute their labor income and inheritance between consumption, investment in education of their children, and saving. Thus, the budget constraint faced by an adult individual belonging to dynasty  $i$  and generation  $t$  is

$$(1 - \tau_w) w_t h_t^i + (1 - \tau_b) b_t^i - \lambda = c_t^i + s_t^i + (1 - s_e) n e_t^i. \quad (2.5)$$

When individuals are old, they receive a return on their saving, which is distributed between consumption and bequests for their children. Therefore, the budget constraint of an old individual of dynasty  $i$  born at period  $t-1$  will be

$$[1 + (1 - \tau_k) r_{t+1}] s_t^i - \theta = x_{t+1}^i + n b_{t+1}^i. \quad (2.6)$$

We also impose the constraint that parents cannot force their children to give them gifts when they (the parents) are old,

$$b_{t+1}^i \geq 0. \quad (2.7)$$

Note also that negative voluntary bequests will never arise in equilibrium given our assumption of one-sided altruism (from parents to children).

Let us assume that the good of this economy is produced by means of the linearly homogeneous net production function  $F(K_t, H_t)$ , where  $K_t$  is the aggregate stock of physical capital and  $H_t$  is the aggregate stock of human capital (or efficiency units of labor) used in period  $t$ . The stock of physical capital fully depreciates after one period. The production function in efficiency units of labor is  $f(z_t)$ , where  $z_t$  stands for the aggregate ratio of physical to human capital,  $K_t/H_t$ . As firms behave competitively, the rental rates of physical and human capital,  $r_t$  and  $w_t$ , are equal to their marginal productivity,

$$1 + r_t = f'(z_t), \quad (2.8)$$

and

$$w_t = f(z_t) - f'(z_t)z_t. \quad (2.9)$$

Because of the small open nature of this economy, the interest rate is fixed at its international level  $r$ . Hence, condition (2.8) determines a constant ratio  $z_t$ , and then (2.9) forces the wage per efficiency unit of labor to be constant as well. Thus,  $r_t = r$ ,  $z_t = z$ , and  $w_t = w$  for all  $t$ .

### 3. The individual problem

In this section, we will solve the problem that a generic individual belonging to dynasty  $i$  and generation  $t$  faces in order to choose the levels of consumption at adult and old ages and the transfers to his immediate descendants. Note first that the amount that an individual receives as inheritance and his level of human capital are the state variables determining his optimal choice.<sup>6</sup> Thus, an individual belonging to dynasty  $i$  and generation  $t$  maximizes (2.2) with respect to  $\{c_t^i, x_{t+1}^i, e_t^i, b_{t+1}^i\}$  subject to (2.3), (2.4), (2.5), (2.6), (2.7) and the non-negative constraints  $c_t^i \geq 0$  and  $x_{t+1}^i \geq 0$ , by taking as given the inheritance received from his parent  $b_t^i$  and his level of human capital  $h_t^i$ . Recall that in this intertemporal maximization problem, the optimal value of the control variable  $e_t^i$  will be either zero or  $\mu$  because of the functional form adopted by the technology producing human capital. Thus, we will solve the individual problem by following a two-stage procedure: first, we take the value of  $e_t^i$  as given, and then solve for the level of saving  $s_t^i$  and bequests  $b_{t+1}^i$ ; and second, we find the optimal level of  $e_t^i$  given the values of  $s_t^i$  and  $b_{t+1}^i$  obtained in the previous stage.

We now proceed by presenting the details of the solution procedure. From the first order conditions of the individual problem, we obtain in Appendix A the following equations:

$$x_{t+1}^i = \rho R(\tau_k) c_t^i, \quad (3.1)$$

and

$$\frac{\beta(1-\tau_b)}{I_{t+1}^i} \leq \frac{n(1+\rho)/R(\tau_k)}{(1-\tau_w)wh_t^i + (1-\tau_b)b_t^i - n(1-s_e)e_t^i - \frac{nb_{t+1}^i}{R(\tau_k)} - \Omega}, \quad (3.2)$$

with

$$\Omega = \lambda + \frac{\theta}{R(\tau_k)},$$

where the condition (3.2) holds with equality if  $b_{t+1}^i > 0$ , and where  $R(\tau_k)$  will denote from now on the after-tax gross rate of return on saving, i.e.,  $R(\tau_k) = 1 + (1-\tau_k)r$ . Equation (3.1) yields the optimal allocation of consumption along the lifetime of an individual belonging to  $i$  born at time  $t-1$ . Equation (3.2) characterizes the optimal level of bequests. This condition tells us that, when the bequest  $b_{t+1}^i$  is positive, the marginal variation in the utility of parents arising from a larger amount of bequests must be equal to zero. On the one hand, the right hand side of this equation is the utility loss experienced by the individual from the decrease in his lifetime income devoted to

<sup>6</sup>Human capital is a state variable because the individuals' education was decided and financed by their parents.

own consumption due to a marginal increase in the amount of bequest left to their children. On the other hand, the left hand side of (3.2) is the utility gain obtained by the individual from the marginal contribution of his bequest to the future lifetime income of their children.

Combining (3.1) with the budget constraints (2.5) and (2.6), we can derive the amount of saving  $s_t^i$  as a function of the amount of intergenerational transfers. Thus, we obtain the following expression:

$$s_t^i = \frac{\rho R(\tau_k) [(1-\tau_w)wh_t^i + (1-\tau_b)b_t^i - n(1-s_e)e_t^i - \lambda] + nb_{t+1}^i + \theta}{(1+\rho)R(\tau_k)}. \quad (3.3)$$

Moreover, from (3.2) we can also compute the optimal level of bequest that parents leave to their children when the constraint (2.7) is not binding, i.e., when  $b_{t+1}^i > 0$ . By taking the condition (3.2) with equality, we directly obtain  $b_{t+1}^i$  as a function of the investment in the education of children  $e_t^i$  and of the endowments  $b_t^i$  and  $h_t^i$ , i.e.,

$$\begin{aligned} b_{t+1}^i &\equiv B(b_t^i, h_t^i, e_t^i) \\ &= \left[ \frac{\beta}{n(1+\beta+\rho)} \right] \left\{ R(\tau_k) [(1-\tau_w)wh_t^i + (1-\tau_b)b_t^i - (1-s_e)ne_t^i - \Omega] \right. \\ &\quad \left. - \left[ \frac{n(1+\rho)(1-\tau_w)}{\beta(1-\tau_b)} \right] w\Delta_{t+1}^i \right\}. \end{aligned} \quad (3.4)$$

In the second stage of our solution procedure we will choose the investment in education  $e_t^i$  that solves the individual's problem. Since the investment in education is indivisible, individuals must actually decide whether they invest  $\mu$  units of income or do not invest at all. Observe that this decision is subject to the following restrictions. First, a positive investment in education for individuals with low levels of income may imply a negative optimal level of bequest, which is not allowed in our economy by assumption. In this case, individuals will not invest in the education of their children. Therefore, the investment in education will be possible only if the individuals' income is sufficiently large so that this investment does not force individuals to leave a negative bequest. Second, if the level of bequest  $b_{t+1}^i$  is positive when the individual invests  $\mu$  units of income in the education of their children, we have to analyze whether this amount  $\mu$  is the optimal level for the investment  $e_t^i$  in education. We next analyze these two issues separately.

#### 3.1. Optimal investment in education

Let us first assume first that the individual has a sufficiently large level of income so that the optimal level of bequest  $b_{t+1}^i$  is positive if he decides to invest in the education of his children. We will now analyze whether to invest in education is an optimal strategy in this case. To this end, we will now check whether to invest the amount  $\mu$  in education is optimal for the individual given the optimal levels of consumptions, saving, and bequest obtained in the previous stage. In Appendix A we obtain that the

optimal levels  $c_t^i$  and  $x_{t+1}^i$  of consumption are given by

$$c_t^i = \left[ \frac{n}{\beta(1-\tau_b)R(\tau_k)} \right] I_{t+1}^i, \quad (3.5)$$

$$x_{t+1}^i = \left[ \frac{n\rho}{\beta(1-\tau_b)} \right] I_{t+1}^i, \quad (3.6)$$

when (2.7) is not binding, i.e., when  $b_{t+1}^i > 0$ . Observe that conditions (3.5) and (3.6) yield the optimal levels  $c_t^i$  and  $x_{t+1}^i$  as increasing functions of  $I_{t+1}^i$ . Therefore, the choice of education investment that maximizes the utility (2.2) at period  $t$  is the one that maximizes the contribution of parents to the future income of their children,  $I_{t+1}^i$ . Since the investment in education is indivisible, individuals must actually decide whether they invest  $\mu$  units of income or do not invest at all. An individual will be willing to invest in the education of their children if and only if this action increases the after-tax lifetime income of their offspring. Thus, in order to determine the optimal decision we must compare the benefit of investing in education with the associated opportunity cost.

Note that an adult individual at period  $t$  can either invest the amount  $\mu$  in the education of his children or save this amount in order to leave a larger bequest in the next period. On the one hand, if he decides to invest in the education of their offspring, he must spend  $(1-s_e)\mu$  units of income per child because  $\mu$  is the cost of education and the government subsidizes the investment in education at the rate  $s_e$ . We obtain from (2.4) that this investment in education raises the after-tax lifetime income of each child by  $(1-\tau_w)\varepsilon w$  units. On the other hand, if that individual decides to save the amount  $(1-s_e)\mu$  in order to make a physical transfer to his children in the next period, then the after-tax lifetime income of the latter will increase by  $(1-\tau_b)(1-s_e)\mu R(\tau_k)$  units since  $R(\tau_k)$  is the after-tax return on saving. Therefore, an individual born at  $t-1$  would like to invest in the education of his children at period  $t$  if and only if the following condition holds:

$$(1-\tau_w)\varepsilon w \geq (1-\tau_b)(1-s_e)\mu R(\tau_k). \quad (3.7)$$

The optimality of investing in education does not depend on the individual's choices, but on the aggregate variables of the economy. We also observe that the optimality of investing in the education of children depends on fiscal policy. From condition (3.7), we directly obtain that the inheritance tax, the capital income tax and the education subsidy raise the willingness of individuals to invest in the education of their direct descendants, whereas the labor income tax reduces this willingness.

When condition (3.7) does not hold, individuals adopt the corner solution  $e_t^i = 0$ . From now on we will assume that condition (3.7) holds. Under this condition individuals will invest in the education of their offspring if they can afford the minimum after-tax cost of education given by  $(1-s_e)\mu$ . Note however that, even when condition (3.7) holds, individuals will not invest in the education of their children if this investment imply a negative optimal level of bequest. In the next subsection we derive the levels of income above which individuals invest in the education of their direct descendants.

### 3.2. Human capital policy

Condition (2.7) is in fact the feasibility condition on investment in education. Given that parents cannot force their children to make transfers to them, they effectively invest in the education of their children if and only if the parents' income is sufficiently large so as to leave a non-negative bequest after making the investment in education. By imposing that parents invest  $\mu$  in the education of their children at period  $t$  (so that  $h_{t+1}^i = 1 + \varepsilon$ ), we obtain from (3.2) that  $b_{t+1}^i > 0$  if and only if the following condition holds:

$$\frac{\beta(1-\tau_b)}{(1-\tau_w)w\varepsilon} > \frac{n(1+\rho)/R(\tau_k)}{(1-\tau_w)wh_t^i + (1-\tau_b)b_t^i - n(1-s_e)\mu - \Omega}. \quad (3.8)$$

This condition says that those parents who have invested in the education of their children but did not leave bequest will obtain a net benefit from leaving a positive amount of bequest. The right hand side of (3.8) is the utility loss experienced by one of these parents arising from the decrease in his lifetime income due to a marginal increase in the amount of bequest left to their children. The left hand side of (3.8) is the utility gain obtained by the aforementioned parent arising from an increase in the the future lifetime income of their children due to a marginal increase in the amount of bequest left to their children.

The feasibility condition (3.8) can be rewritten as a threshold level for the inheritance  $b_t^i$  received by the parents. This threshold is determined by the human capital level of the parents  $h_t^i$ . On the one hand, if the parents are non-educated (i.e.,  $h_t^i = 1$ ), then the threshold level for bequests is

$$\tilde{b} = \left( \frac{1}{1-\tau_b} \right) \left\{ (1-s_e)n\mu + \left[ \frac{n(1+\rho)\varepsilon}{\beta(1-\tau_b)R(\tau_k)} - 1 \right] (1-\tau_w)w + \Omega \right\}. \quad (3.9)$$

Thus, an individual with a level of human capital  $h_t^i = 1$  effectively invests in the education of his offspring if and only he has received an inheritance  $b_t^i$  that satisfies  $b_t^i \geq \tilde{b}$ . On the other hand, if parents are educated (i.e.,  $h_t^i = 1 + \varepsilon$ ), then the threshold level for bequest in this case is given by

$$\hat{b} = \left( \frac{1}{1-\tau_b} \right) \left\{ (1-s_e)n\mu + \left[ \frac{n(1+\rho)\varepsilon}{\beta(1-\tau_b)R(\tau_k)} - (1+\varepsilon) \right] (1-\tau_w)w + \Omega \right\}. \quad (3.10)$$

Thus, an individual with a level of human capital  $h_t^i = 1 + \varepsilon$  effectively invests in the education of their children if and only if his inheritance  $b_t^i$  satisfies  $b_t^i \geq \hat{b}$ .

The threshold levels (3.9) and (3.10) of bequest were obtained by eliminating those situations where a positive investment in the education of children would imply a positive transfer from children to parents. Those threshold values determine the dynamics of the human capital level within each dynasty. In particular, the dynamics of human capital inside a dynasty is given by the following dynamic equation:

$$h_{t+1}^i = \begin{cases} 1 & \text{if either } h_t^i = 1 \text{ and } b_t^i < \tilde{b} \text{ or } h_t^i = 1 + \varepsilon \text{ and } 0 \leq b_t^i < \hat{b}; \\ 1 + \varepsilon & \text{if either } h_t^i = 1 \text{ and } b_t^i \geq \tilde{b} \text{ or } h_t^i = 1 + \varepsilon \text{ and } b_t^i \geq \hat{b}. \end{cases} \quad (3.11)$$

By comparing (3.10) and (3.9), we directly obtain that  $\tilde{b} > \hat{b}$ . Since the labor income of the parents who are educated is larger than the labor income of the non-educated

parents, the first type of parents need a smaller amount of inheritance to afford the education of their children.

In our economy, when individuals do not invest in the education of their offspring, they always leave a strictly positive amount of bequest. This follows from the fact that the utility function (2.2) satisfies the Inada condition at origin with respect to the parents contribution to the lifetime income of their children  $I_{t+1}^i$ , that is, the marginal utility with respect to  $I_{t+1}^i$  goes to infinity when this contribution tends to zero.

From the threshold levels of bequests defined in this section and equation (3.4), we get the following equation characterizing the dynamics of bequests within a dynasty:

$$b_{t+1}^i = \begin{cases} B^1(b_t^i) & \text{if } h_t^i = 1 \text{ and } 0 \leq b_t^i < \tilde{b}; \\ B^2(b_t^i) & \text{if } h_t^i = 1 \text{ and } b_t^i > \tilde{b}; \\ B^3(b_t^i) & \text{if } h_t^i = 1 + \varepsilon \text{ and } 0 \leq b_t^i < \hat{b}; \\ B^4(b_t^i) & \text{if } h_t^i = 1 + \varepsilon \text{ and } b_t^i > \hat{b}; \end{cases} \quad (3.12)$$

where  $B^1(b_t^i) \equiv B(b_t^i, 1, 0)$ ,  $B^2(b_t^i) \equiv B(b_t^i, 1, \mu)$ ,  $B^3(b_t^i) \equiv B(b_t^i, 1 + \varepsilon, 0)$ , and  $B^4(b_t^i) \equiv B(b_t^i, 1 + \varepsilon, \mu)$ .

The dynamic equations (3.11) and (3.12) determine the policy functions for human capital and bequests, respectively, within a dynasty when condition (3.7) holds. In other words, these two equations determine the level of human capital and bequests for the next cohort of the dynasty given the human capital and bequest of the present cohort.

#### 4. The dynamics of dynastic income

In this section we study the dynamics of the joint distribution of bequests and human capital. Under our assumptions, this dynamics follows directly from the dynamic equations of bequests and human capital (3.11) and (3.12), respectively. Since we have considered a small open economy, the evolution of each dynasty does not depend on the aggregate distribution. Thus, in this section we analyze the evolution of bequests and human capital for a given dynasty along time. In this sense, observe that individuals within a cohort differ in two respects: first, individuals have different levels of income in their second period of life since they have received different transfers from their parents; and, second, individuals also differ in the composition of income due to the different composition of the transfers received from their parents. Thus, the amount that a dynasty initially receives as bequest and the initial level of human capital fully determine the entire posterior path of bequests, human capital, and income.

##### 4.1. Stationary distribution of income

We will now characterize the stationary distribution of bequests and human capital. For that purpose, we will prove that the dynamic system composed of equations (3.11) and (3.12) has at most two stationary solutions. We will see that there are three candidates

for these steady states: a corner solution, where bequests are zero; and two interior solutions given by the two possible fixed points of (3.12), which we will denote by  $\bar{b}^1$  and  $\bar{b}^2$ . The point  $\bar{b}^1$  is a fixed point of  $B^1(b_t^i)$ , whereas  $\bar{b}^2$  is a fixed point of  $B^4(b_t^i)$ . Thus, we get from (3.4) that

$$\bar{b}^1 = \frac{\beta R(\tau_k) [(1 - \tau_w) w - \Omega]}{n(1 + \beta + \rho) - \beta R(\tau_k)(1 - \tau_b)}, \quad (4.1)$$

and

$$\bar{b}^2 = \frac{\beta R(\tau_k) \left[ (1 - \tau_w) w (1 + \varepsilon) - (1 - s_e) n \mu - \frac{n(1 + \rho)(1 - \tau_w) w \varepsilon}{\beta R(\tau_k)(1 - \tau_b)} - \Omega \right]}{n(1 + \beta + \rho) - \beta R(\tau_k)(1 - \tau_b)}. \quad (4.2)$$

Obviously, a necessary condition for a level of bequests being an interior steady state is that an educated (non-educated) parent who has received this level of inheritance does (not) actually invest in the education of their children. We will see next that the fixed points of the functions  $B^2(b_t^i)$  and  $B^3(b_t^i)$  can not be stationary values of bequest. In order to prove that a fixed point  $\bar{b}$  of  $B^2(b_t^i)$  is not a steady state for bequests, let us assume that  $b_t^i = \bar{b} > \tilde{b}$  and  $h_t^i = 1$ . As follows from (3.11) and (3.12), this individual leaves a bequest per capita equal to  $b_{t+1}^i = B^2(\bar{b}) = \bar{b}$  and invests in the education of their children so that  $h_{t+1}^i = 1 + \varepsilon$ . Thus, a son of the previous individual will enjoy an endowment vector  $(h_{t+1}^i, b_{t+1}^i)$  equal to  $(1 + \varepsilon, \bar{b})$  so that he will also invest in the education of their children and will leave them a bequest equal to  $b_{t+2}^i = B^4(\bar{b}) \neq \bar{b}$ . This proves that the fixed point of  $B^2(b_t^i)$  is not a steady state because it is not a rest point of the dynamic equation (3.11).

We can follow similar arguments to prove that a fixed point  $\bar{b}$  of  $B^3(b_t^i)$  cannot be a steady state. For this purpose, assume that  $b_t^i = \bar{b} < \hat{b}$  and  $h_t^i = 1 + \varepsilon$ . As follows from (3.11) and (3.12), this individual leaves a bequest per capita equal to  $b_{t+1}^i = B^3(\bar{b}) = \bar{b}$ . However, he does not invest in the education of their children so that  $h_{t+1}^i = 1$ . Thus, a son of the previous individual will enjoy an endowment  $(h_{t+1}^i, b_{t+1}^i)$  equal to  $(1, \bar{b})$  and, thus, he will not invest either in the education of their children and will leave them a bequest equal to  $b_{t+2}^i = B^1(\bar{b}) \neq \bar{b}$ . This proves in turn that the fixed point of  $B^3(b_t^i)$  is not a rest point of the dynamic equation (3.11).

As a summary, we conclude that the mobility in human capital across generations prevents the fixed points of  $B^2(b_t^i)$  and  $B^3(b_t^i)$  from being steady states for bequests. However, by the same reason, the fixed points of  $B^1(b_t^i)$  and  $B^4(b_t^i)$  may be steady states. In other words, the level of bequest  $\bar{b}^1$  is stationary because those non-educated individuals who have received this level of inheritance do not invest in the education of their children, whereas  $\bar{b}^2$  is a stationary level of bequests since the educated individuals who have received this level of inheritance do finance the education of their offspring.

Observe that  $\bar{b}^1$  can be either smaller or larger than  $\bar{b}^2$ . In the first case the educated individuals leave a larger amount of bequests to their children than the non-educated, whereas the opposite is true in the second case. By using (4.1) and (4.2), we obtain that  $\bar{b}^1 < \bar{b}^2$  if and only if

$$(1 - \tau_w) w \varepsilon - (1 - s_e) n \mu - \frac{n(1 + \rho)(1 - \tau_w) w \varepsilon}{\beta R(\tau_k)(1 - \tau_b)} > 0. \quad (4.3)$$

The left-hand side of (4.3) collects the three forces driving the relationship between  $\bar{b}^1$  and  $\bar{b}^2$ . This relationship depends first on how large is the labor income of educated parents with respect to the income of non-educated parents (the education premium). Second, the education cost reduces the level of bequest that the parents investing in education are willing to leave to their children. Finally, the larger is the contribution of education to the labor income of children, the smaller is the amount of bequest that educated parents must leave to achieve the optimal amount of the contribution to the future income of their children.

In order to simplify the exposition and keep the length of the paper within a reasonable bound, we will only analyze the dynamics of the more empirically plausible case where  $\bar{b}^1 < \bar{b}^2$ . For instance, Nordblom and Ohlsson (2005) estimate that the education level of parents in Sweden increases the probability that they transfer both human and physical capital to their children. That is, intergenerational transfers of human capital and physical wealth are complements. Therefore, we will assume that (4.3) holds from now on.<sup>7</sup>

We are interested in those parameter configurations for which the interior steady states for bequests  $\bar{b}^1$  and  $\bar{b}^2$  exist and are stable. In this case, the economy exhibits heterogeneity among individuals at the steady state and, thus, we can analyze how the initial composition of the intergenerational transfers and the fiscal policy parameters affect income inequality and human capital mobility. In order to state the existence and stability of  $\bar{b}^1$  and  $\bar{b}^2$ , we must impose some assumptions on the fundamentals of our economy. In particular, the existence and stability of the stationary bequests depend on whether the non-negative constraint on bequests (2.7) is binding.

First, the existence of two interior and stable fixed points  $\bar{b}^1$  and  $\bar{b}^2$  requires the functions  $B^j(b_i^j)$  in (3.12) to have slope smaller than one for all  $j = 1, 2, 3, 4$ , and to satisfy that  $B^1(0) > 0$  and  $B^4(0) > 0$ . On the one hand, as was pointed in the previous section, the Inada condition of the utility function (2.2) with respect to  $I_{i+1}^j$  ensures that  $B^1(b_i^j) > 0$  for all  $b_i^j \geq 0$ . Moreover, the condition (4.3) ensures that  $B^4(b_i^j) > B^1(b_i^j)$  for all  $b_i^j \geq 0$  so that, in this case, it is also true that  $B^4(0) > 0$ . On the other hand, given these properties, interior fixed points exist if and only if the functions  $B^j(b_i^j)$  have slope smaller than one. This property of  $B^j(b_i^j)$  holds under the following condition:

$$\frac{\beta(1 - \tau_b)R(\tau_k)}{n(1 + \beta + \rho)} < 1, \quad (4.4)$$

which also ensures the stability of the interior steady states  $\bar{b}^1$  and  $\bar{b}^2$  provided they exist.

Second, since condition (4.4) holds, the interior steady states  $\bar{b}^1$  and  $\bar{b}^2$  exist if and only if the threshold values  $\tilde{b}$  and  $\hat{b}$  are positive and negative, respectively. On the one hand, if  $\tilde{b} < 0$  then the function  $B^1(b_i^j)$  is not defined for positive values of  $b_i^j$  and, thus, the steady state  $\bar{b}^1$  does not exist, which means that the non-educated individuals always decide to invest in the education of their children in this case. On the other hand, if  $\hat{b} > 0$  then the function  $B^4(b_i^j)$  takes values smaller than one since it has slope smaller than one and, hence, the steady state  $\bar{b}^2$  does not arise. The following two

<sup>7</sup>The analysis of the case with  $\bar{b}^1 > \bar{b}^2$  becomes just a mechanical exercise that replicates the same arguments that we will use in the rest of the paper for the case under consideration.

conditions ensure that  $\tilde{b} > 0$  and  $\hat{b} < 0$ , respectively:

$$(1 - s_e)n\mu + \left[ \frac{n(1 + \rho)\varepsilon}{\beta(1 - \tau_b)R(\tau_k)} - 1 \right] (1 - \tau_w)w + \Omega > 0, \quad (4.5)$$

and

$$(1 - s_e)n\mu + \left[ \frac{n(1 + \rho)\varepsilon}{\beta(1 - \tau_b)R(\tau_k)} - (1 + \varepsilon) \right] (1 - \tau_w)w + \Omega < 0. \quad (4.6)$$

Finally, condition (4.5) guarantees that  $\bar{b}^1 > 0$  but is not sufficient to ensure that this fixed point of (3.4) is a steady state for the amount of bequest. In order to do so, we also need to impose that

$$\bar{b}^1 < \tilde{b}. \quad (4.7)$$

If (4.7) does not hold, the level of bequest  $\bar{b}^1$  could not be a steady state because the function  $B^1(b_i^j)$  does not characterize the dynamics of bequest left by non-educated individuals who have received an inheritance larger than  $\tilde{b}$ . Given the definitions of  $\tilde{b}$  and  $\bar{b}^1$  in (3.9) and (4.1), respectively, the equation (4.7) ends up being just a condition on the fundamentals of the economy.

From now on we will also assume that the conditions (4.4), (4.5), (4.6) and (4.7) hold. Under these conditions, our economy converges to a two-point distribution with appealing empirical properties.<sup>8</sup> Under this distribution some dynasties leave a bequest to each of their children equal to  $\bar{b}^1$  and do not invest in their education, whereas other dynasties do invest in the education of their children and leave a positive bequest per capita equal to  $\bar{b}^2$ .<sup>9</sup> This is the case depicted in Figure 1, which plots the relationship between the bequests left to children and the inheritance received from parents given by the dynamic equation (3.12). Note that this relationship is piecewise linear.

[Insert Figure 1]

The previous two-point stationary distribution has the property that the educated individuals who invest in the education of their children leave larger bequests to their children than individuals who do not invest in education. As we have already mentioned, this property agrees with the empirical evidence provided by Nordblom and Ohlsson (2005).

The initial distribution of bequests and human capital determines both the stationary income distribution and the intergenerational mobility in human capital. This dependence arises in our economy because the initial distribution of bequests and human capital drives the dynamics followed by the composition of intergenerational transfers. In order to illustrate this point, we will characterize in the following subsection the entire path of bequests and human capital for all types of dynasties when the economy converges to the aforementioned two-point distribution. We will use this analysis to study in the next section the impact of fiscal policy in the distribution dynamics for a given initial distribution.

<sup>8</sup>In the case with  $\bar{b}^1 > \bar{b}^2$  (i.e., when (4.3) does not hold), we must also impose that  $B^4(0) > 0$  to ensure the existence of a two-point interior stationary distribution.

<sup>9</sup>When at least one of these conditions does not hold, then the economy converges to either a degenerate distribution or a distribution defined by corner steady state values of bequest. In Appendix B we present all possible configurations of the stationary distribution.

## 4.2. Dynamic analysis

In this subsection we analyze the dynamics of an economy that converges to the previous two-point distribution. As was shown in the previous subsection, this occurs when the conditions (4.3), (4.4), (4.5) and (4.6) hold. We will next prove that the initial distribution of bequests and human capital determines the number of dynasties converging to a situation with  $h_t^i = 1$  and  $b_t^i = \bar{b}^1$  and those converging to  $h_t^i = 1 + \varepsilon$  and  $b_t^i = \bar{b}^2$ .

We first observe that those dynasties whose members have the initial human capital  $h_0^i = 1$  and who have received an inheritance  $b_0^i$  smaller than  $\tilde{b}$  converge to a steady state given by  $h^i = 1$  and  $b^i = \bar{b}^1$ , whereas all the dynasties with members having the initial human capital  $h_0^i = 1 + \varepsilon$  converge to a steady state given by  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$ . This conclusion directly follows from the dynamics of human capital and bequests described by (3.11) and (3.12), and after using condition (4.4) ensuring the stability of the steady states  $\bar{b}^1$  and  $\bar{b}^2$ . On the one hand, the members of a dynasty with  $h_t^i = 1$  and  $b_t^i$  smaller than  $\tilde{b}$  do not invest in the education of their children and leave bequests satisfying  $b_{t+1}^i < \tilde{b}$ . On the other hand, the members of a dynasty with a level of human capital  $h_t^i = 1 + \varepsilon$  always invest in the education of their children because condition (4.6) ensures that  $\hat{b} < 0$ . Condition (4.4) ensures that the bequests of the former and the latter dynasties will converge to  $\bar{b}^1$  and  $\bar{b}^2$ , respectively.

Less trivial is the dynamic adjustment of human capital and bequests for the other group of dynasties, i.e., those with  $h_0^i = 1$  and  $b_0^i$  larger than  $\tilde{b}$ . The members of these dynasties decide to finance the education of their children as dictated by equation (3.11). However, it is necessary to know how large is the amount of bequests  $b_{t+1}^i = B^2(b_t^i)$  that they leave since this amount determines in turn the behavior of their offspring. From condition (4.6), they always leave a bequest per capita  $b_{t+1}^i$  larger than  $\tilde{b}$ . The children of those individuals will then decide to leave a bequest per capita equal to  $b_{t+2}^i = B^4(b_{t+1}^i)$  and, thus, the dynasty will converge to the steady state given by  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$ . Therefore, the threshold value of bequest  $\tilde{b}$  determines the dynamics of the initially non-educated dynasties and, in particular, their human capital mobility. The non-educated dynasties will converge to the steady state associated with  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$  if their members are initially endowed with an inheritance  $b_0^i$  larger than  $\tilde{b}$ , whereas these non-educated dynasties with an inheritance  $b_0^i$  smaller than  $\tilde{b}$  will converge to the steady state given by  $h^i = 1$  and  $b^i = \bar{b}^1$ .

We can thus summarize the dynamic behavior of the economy considered in this subsection as follows. The dynasties with an initial level of human capital  $h_0^i = 1$  will converge: (i) to the steady state  $h^i = 1$  and  $b^i = \bar{b}^1$  if  $b_0^i < \tilde{b}$ ; and (ii) to the steady state  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$  if  $b_0^i > \tilde{b}$ . The dynasties with an initial level of human capital  $h_0^i = 1 + \varepsilon$  will always converge to the steady state  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$ . Therefore, we have shown that the initial distribution of bequests and human capital determines the stationary distribution of these variables, and thus the stationary distribution of income. In particular, our model predicts that the propensity to invest in the education of children positively depends on the education level of parents.

With respect to human capital mobility, we observe that only the non-educated dynasties may experience intergenerational mobility. In this case, the inheritance is the variable that determines this mobility. In particular, the dynasties with an initial

level of human capital  $h_0^i = 1$  educate their children in the first period if  $b_0^i > \tilde{b}$ , and then they remain as educated dynasties forever. Therefore, for these dynasties the human capital adjusts instantaneously to the level  $1 + \varepsilon$  and the amount of bequest converges monotonously along time to the steady state  $\bar{b}^2$ . Moreover, the threshold  $\tilde{b}$  in (3.9) contains all the information about the determinants of intergenerational mobility. In the next section, we will analyze how fiscal policy affects this mobility.

## 5. Effects of fiscal policy on the stationary distribution

In this section we will analyze how fiscal policy affects the stationary distribution towards which an economy converges given an initial distribution. We will assume a parametric configuration ensuring that the economy converges to the empirically plausible two-point distribution considered in Subsection 4.2. Given an initial distribution of bequests and human capital, we will analyze how non-anticipated permanent marginal shocks on the fiscal parameters alter this stationary distribution. In particular, we will develop balanced-budget incidence analyses where the public consumption will accommodate the permanent fiscal shocks in order to satisfy the constraint (2.1). Observe that this assumption implies that the fiscal reforms do not have ex-ante redistribution effects. However, the proposed reforms may have ex-post distribution effects since fiscal policy distorts the optimal decisions taken by each individual.

Fiscal policy can alter the stationary levels of bequests  $\bar{b}^1$  and  $\bar{b}^2$ , and it can also affect the proportion of dynasties converging to each of these steady states by distorting the intergenerational mobility in human capital. The distance between  $\bar{b}^1$  and  $\bar{b}^2$  measures the inequality between the income of educated and non-educated adult individuals due to the different amount of inheritance they receive. To obtain the effects on  $\bar{b}^1$  and  $\bar{b}^2$  we must analyze the impact of fiscal policies on the functions  $B^1(b_t^i)$  and  $B^4(b_t^i)$ , respectively.<sup>10</sup> The effect of these policies on intergenerational mobility in human capital is given instead by their impact on the threshold level of bequests  $\tilde{b}$ . A small value of  $\tilde{b}$  means that the set of initial amounts of bequest for which non-educated dynasties end up being educated becomes larger. In other words, a small value of  $\tilde{b}$  makes easier intergenerational mobility.<sup>11</sup>

We next study separately the effects of each of the tax instruments under consideration.

### 5.1. Inheritance taxation

We analyze the effects of a fiscal reform consisting on a marginal increase in the tax rate on inheritances so that the change in fiscal revenue is allocated to public consumption. Under this condition, we show that the tax on inheritance reduces the value of the stationary bequests  $\bar{b}^1$  and  $\bar{b}^2$ . To prove this conclusion, we differentiate the function

<sup>10</sup>In fact, the effect of fiscal policies on  $\bar{b}^1$  and  $\bar{b}^2$  can also be directly derived by applying respectively the implicit function theorem to the equation  $\bar{b}^1 = B^1(\bar{b}^1)$  and  $\bar{b}^2 = B^4(\bar{b}^2)$  and using Assumption (4.4).

<sup>11</sup>The fiscal policy can also affect the threshold level  $\tilde{b}$ . However, since we will only consider marginal shocks in fiscal policy, we can maintain the assumption that this threshold level remains negative after the shocks.

$B(b_t^i, h_t^i, e_t^i)$  with respect to  $\tau_b$ . From (3.4), we obtain that

$$\frac{\partial B(b_t^i, h_t^i, e_t^i)}{\partial \tau_b} = - \left[ \frac{\beta R(\tau_k) b_t^i}{n(1 + \beta + \rho)} + \frac{(1 + \rho)(1 - \tau_w) w \Delta_{t+1}^i}{n(1 + \beta + \rho)(1 - \tau_b)^2} \right], \quad (5.1)$$

which is negative. Since  $\bar{b}^1$  and  $\bar{b}^2$  are fixed points of  $B^1(b_t^i) = B(b_t^i, 1, 0)$  and  $B^4(b_t^i) = B(b_t^i, 1 + \varepsilon, \mu)$ , respectively, the effects of the inheritance tax on these stationary solutions immediately follow from (5.1) and (2.4). Moreover, the marginal increase in the rate of this tax reduces the gap between  $\bar{b}^1$  and  $\bar{b}^2$  because the negative impact of this permanent policy shock on  $B^4(b_t^i)$  is larger than in  $B^1(b_t^i)$  as follows from (5.1) after substituting the corresponding value of  $\Delta_{t+1}^i$ . Thus, the increase in the tax rate on inheritances reduces the stationary differences in the income per capita between educated and non-educated individuals.

We now analyze how the marginal permanent shock in  $\tau_b$  affects the stationary distribution. For that purpose, we study the effects of the tax on the threshold level of bequests  $\tilde{b}$  that determines the steady state towards which each dynasty converges. From equation (3.9), we directly obtain that

$$\frac{\partial \tilde{b}}{\partial \tau_b} = \frac{\tilde{b}}{(1 - \tau_b)} + \frac{n(1 + \rho)(1 - \tau_w) w \varepsilon}{\beta(1 - \tau_b)^3 R(\tau_k)},$$

which is positive, i.e., the marginal increase in  $\tau_b$  pushes the level of  $\tilde{b}$  up. Therefore, the tax on inheritances reduces the number of initially non-educated dynasties converging to the steady state given by  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$ . As was expected from the dynamic analysis of the previous section, this tax does not alter the number of initially educated dynasties converging to the steady state given by  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^1$ .

The effects of inheritance taxation on the stationary distribution can then be summarized as follows. An increase in the rate of this tax raises the fraction of individuals converging to the steady state given by  $h^i = 1$  and  $b^i = \bar{b}^1$ . Thus, the stationary fraction of non-educated individuals increases and the level of bequests becomes smaller in the long run. This reduction in the level of bequests is larger for the group of educated adult individuals (i.e., for the richest people). The aggregate adult income at the steady state then goes down and, moreover, the proportion of aggregate income enjoyed by the poorest adult individuals rises. Therefore, all these effects of the inheritance taxation translate into a reduction in the inequality between educated and non-educated people at the stationary distribution of income at the cost of a reduction in aggregate income. Figure 2 presents the change in the relative frequencies of the stationary distribution of bequests after an increase in the tax rate on inheritances.

[Insert Figure 2]

The economic intuition of these results follows directly from the optimality condition (3.2). The inheritance tax reduces the overall benefit that parents obtain from leaving bequest to their children. On the one hand, the increase in the rate of this tax reduces the disposable income of parents. Hence, the tax increases the utility loss derived from the decrease in the lifetime income devoted to own consumption due to the marginal increase in the amount of bequest left to their children. This first effect of the tax on

the bequest's decision margin is given by the first term of the derivative (5.1). On the other hand, this tax affects the utility gain that parents obtain from the contribution of their bequest to the lifetime income of their offspring depending on whether or not they have invested in the education of their children. In particular, the tax reduces this utility gain when parents invest in education. This effect is given by the second term of the derivative (5.1). This asymmetric effect of the inheritance tax on the utility gain comes from the fact that this tax may also distort the composition of intergenerational transfers. Although the inheritance tax reduces the inheritance received by individuals, this tax does not affect the before-tax transfer received from their parents. Hence, the inheritance tax should not affect the level of bequests that parents decide to leave to their children. However, this tax does reduce the marginal contribution of bequest to the lifetime income of children relative to the marginal contribution of education.

Summarizing, the inheritance tax makes bequests less attractive as an instrument to increase the lifetime income of children. This explains why the inheritance tax reduces the amount of bequest that parents leave to their offspring. Moreover, this distortion of the tax on the bequest's decision margin also explains why this tax raises the amount of inheritance that an individual must receive from his parents in order to be willing to invest in the education of their children.

Given the effect of the inheritance taxation on the stationary level of bequests and on the stationary distribution of dynasties, we can derive the effect of this tax on the stationary amount of saving. From (3.3), and using (5.1), we get that

$$\frac{\partial s_t^i}{\partial \tau_b} = - \frac{\rho R(\tau_k) b_t^i - n \left[ \frac{\partial B(b_t^i, h_t^i, e_t^i)}{\partial \tau_b} \right]}{(1 + \rho) R(\tau_k)},$$

which is negative. Thus, we conclude that in our small open economy, inheritance taxes reduce the saving of each individual at the steady state. Since in our economy leaving bequests is a motive for saving, a tax on inheritances affects negatively savings and bequests. Obviously, from this result and from the fact that the stationary proportion of non-educated individuals raises, we can conclude that an inheritance tax affects negatively the aggregate amount of saving at the steady state.

## 5.2. Labor income taxation

By following the same procedure as in the previous subsection, we obtain that the effect of labor income taxation. We now consider a balanced-budget reform consisting of a marginal increase in the rate of this tax, such that the budget constraint (2.1) is satisfied. Under this assumption, we first get from (3.4) that

$$\frac{\partial B(b_t^i, h_t^i, e_t^i)}{\partial \tau_w} = \left[ \frac{\beta w R(\tau_k)}{n(1 + \beta + \rho)} \right] \left[ \frac{n(1 + \rho) \Delta_{t+1}^i}{\beta(1 - \tau_b) R(\tau_k)} - h_t^i \right]. \quad (5.2)$$

By using (2.4) and (3.12), we get that  $\frac{\partial B^1}{\partial \tau_w} < 0$  and  $\frac{\partial B^4}{\partial \tau_w} < 0$ , where the last inequality follows from condition (4.3). Hence, a marginal increase in the rate of the labor income tax reduces the stationary level of bequests  $\bar{b}^1$  and  $\bar{b}^2$ . Moreover, by using condition (4.3) we also show that an increase in the rate of the labor income tax pushes the gap between  $\bar{b}^1$  and  $\bar{b}^2$  down. In other words, the labor income taxation reduces the

stationary differences in the income per capita between educated and non-educated individuals.

We now analyze how a marginal increase in  $\tau_w$  affects the mobility in human capital. From (3.9), we get that

$$\frac{\partial \tilde{b}}{\partial \tau_w} = \left( \frac{w}{1 - \tau_b} \right) \left[ 1 - \frac{n(1 + \rho)\varepsilon}{\beta(1 - \tau_b)R(\tau_k)} \right].$$

Let us define

$$\varepsilon^* = \frac{\beta(1 - \tau_b)R(\tau_k)}{n(1 + \rho)}. \quad (5.3)$$

If  $\varepsilon < \varepsilon^*$  then  $\frac{\partial \tilde{b}}{\partial \tau_w} > 0$ , whereas  $\frac{\partial \tilde{b}}{\partial \tau_w} < 0$  when  $\varepsilon > \varepsilon^*$ . Therefore, the labor income tax raises the number of initially non-educated dynasties converging to the steady state given by  $h^i = 1 + \varepsilon$  and  $b^i = \tilde{b}^2$  if and only if the *education premium*  $\varepsilon$  is sufficiently large. In this case, a positive permanent shock in the tax rate raises the stationary fraction of population that is educated ( $h^i = 1 + \varepsilon$ ) and leave bequests equal to  $\tilde{b}^2$ . Clearly, the opposite conclusion is derived from the case with a education premium below the threshold  $\varepsilon^*$ . In this case, the labor income tax has the same effect as the inheritance tax. Figure 2 and 3 illustrates the change in the relative frequencies of the stationary distribution of bequests after an increase in the tax rate on labor income.

[Insert Figure 3]

Again the intuition of these effects of labor income tax can be easily obtained from the condition (3.2). There are two countervailing forces that give rise to these effects. On the one hand, the labor tax raises the marginal utility loss experienced by the parents who leave bequests to their children because the tax reduces the disposable lifetime income of parents. This negative effect of the tax on the bequest's decision margin is given by the derivative of the right-hand side of (3.2). Observe that the larger the inheritance  $b_t^i$  received by the parent, the smaller this negative effect of labor income tax. On the other hand, if parents invest in the education of their children, then the labor income tax reduces the after-tax contribution of parents to the lifetime income of their children. We observe that this distortion raises the marginal utility gain that those parents investing in education derive from the contribution of their bequests to the lifetime income of their children. Clearly, from (3.2) we obtain that

$$\frac{\partial \pi_t^i}{\partial \tau_w} = \frac{\beta(1 - \tau_b)w\Delta_{t+1}^i}{(I_{t+1}^i)^2} \quad (5.4)$$

where  $\pi_t^i$  is the marginal utility gain of leaving bequest given by the left-hand side of (3.2). The previous derivative is positive in the case of parents who invest in education ( $\Delta_{t+1}^i = \varepsilon$ ), whereas the derivative is equal to zero in the case of parents who do not invest in the education of their children ( $\Delta_{t+1}^i = 0$ ).

We have proved from (5.2) that the overall effect of the labor income tax on the stationary level of bequests is negative for both the educated and the non-educated individuals. Evidently, this net effect depends on the education premium  $\varepsilon$ . Observe that this premium determines by how much a parent who invest in the education of

their children contributes to their lifetime income. Hence, this premium affects the marginal utility gain derived from investing in education. In particular, from (5.4) we get

$$\frac{\partial^2 \pi_t^i}{\partial \tau_w \partial \varepsilon} = \frac{\beta(1 - \tau_b)w[(1 - \tau_b)b_{t+1}^i - (1 - \tau_w)\Delta_{t+1}^i]}{(I_{t+1}^i)^3}. \quad (5.5)$$

While the sign of the previous derivative is ambiguous, we have proved that the overall effect of labor income tax on the stationary bequests is negative for all values of  $\varepsilon$ . However, the derivative (5.5) still contains information about how the education premium  $\varepsilon$  affects the stationary fraction of population that becomes educated ( $h^i = 1 + \varepsilon$ ). By evaluating this derivative for the individual who has received an inheritance  $b_t^i = \tilde{b}$ , we can determine the effect of  $\varepsilon$  on the stationary distribution. To this end, we evaluate the derivative (5.5) at  $\Delta_{t+1}^i = 0$  because that individual is indifferent between investing or not in the education of their children. For this marginal individual, we clearly get that the derivative (5.5) is positive. Hence, the positive effect of the labor income tax on the marginal utility gain from leaving bequests is an increasing function of  $\varepsilon$  for parents who have received an inheritance  $b_t^i = \tilde{b}$ . Therefore, if the education premium is sufficiently large ( $\varepsilon > \varepsilon^*$ ), then the tax raises the willingness to invest in the education of their children for those individuals. In other words, when  $\varepsilon > \varepsilon^*$  those marginal individuals need a smaller inheritance to invest in education without affecting the transfer to their children. This mechanism explains why the labor income tax raises the stationary fraction of educated population when the education premium is sufficiently large.

At this point, one should investigate what are the empirically plausible value for the education premium  $\varepsilon$ . Note that the condition (4.3) implies that  $\varepsilon^*$  is larger than unity. Thus, in an economy with  $\varepsilon > \varepsilon^*$  the educated individuals obtain a wage per efficiency unit of labor that is more than twice as much the marginal wage perceived by the non-educated people. Several empirical studies provide evidence supporting the existence of a large education premium (see, e.g., Bound and Johnson, 1992; or Barro and Lee, 2000), and show an dramatic increase in this premium from the middle of the past century (see, e.g., Autor, 1998). For instance, Barro and Lee (2000) estimate that the wage of individuals who have completed the higher level of education relative to those with an incomplete primary level is around 2.18.

Finally, we can see that labor income taxation reduces individual saving at the steady state of both educated and non-educated individuals. We obtain this result from differentiating (3.3) and using the derivative (5.2). The effect of this tax on the amount of aggregate saving then depend on the education premium because  $\varepsilon$  determines whether the proportion of educated individuals goes up or down. If  $\varepsilon < \varepsilon^*$  the aggregate saving depends negatively on the tax rate because in this case the tax raises the proportion of non-educated individuals. However, the effect of the tax on aggregate saving is ambiguous when  $\varepsilon > \varepsilon^*$  because in this case the proportion of educated individuals goes up and the saving of the two types of individuals decrease.

### 5.3. Capital income taxation

We now analyze the effects of capital income taxation by considering a balanced-budget reform as in the preceding subsections. We show that this capital income tax has

the same marginal effects on the stationary levels of bequests and on the stationary distribution of human capital as the tax on inheritance analyzed in Subsection 5.1. On the one hand, we get from (3.4) that

$$\frac{\partial B(b_t^i, h_t^i, e_t^i)}{\partial \tau_k} = -\frac{\beta r [(1 - \tau_w)wh_t^i + (1 - \tau_b)b_t^i - n(1 - s_e)e_t^i - \lambda]}{n(1 + \beta + \rho)}. \quad (5.6)$$

Observe that the expression inside the bracket in (5.6) is the individual's disposable income at the adult age (see the budget constraint (2.5)). Hence, the derivative (5.6) is negative. Therefore, the tax on capital income reduces the stationary levels of bequests  $\bar{b}^1$  and  $\bar{b}^2$  and, moreover, this reduction is larger for the amount of bequests  $\bar{b}^2$  of the educated individuals.

On the other hand, the marginal increase in the tax rate  $\tau_k$  raises the threshold level of bequests  $\tilde{b}$ . Clearly, we obtain from (3.9)

$$\frac{\partial \tilde{b}}{\partial \tau_k} = \frac{n(1 + \rho)r(1 - \tau_w)w\varepsilon}{\beta [(1 - \tau_b)R(\tau_k)]^2} + \frac{\theta r}{(1 - \tau_b)[R(\tau_k)]^2}, \quad (5.7)$$

which is positive. This means that this fiscal reform leads to an increase in the fraction of individuals converging to the steady state given by  $h^i = 1$  and  $b^i = \tilde{b}^1$ . Thus, the non-educated adult individuals increase their stationary weight in the total population. Therefore, the initial aggregate income of individuals at the steady state goes down and, moreover, the proportion of initial aggregate income enjoyed by the poorest adult individuals rises. Figure 2 depicts also the effect of a rise in the capital income tax.

The condition (3.2) also provides the economic mechanism underlying the previous effects of capital income taxation. Note that this tax reduces the value at the old age of the individual's lifetime income disposable for consumption for a given level of bequest left to children. Hence, the tax increases the marginal utility loss derived from reducing consumption to increase the amount of bequest left to children. However, the tax does not alter the contribution of parents to the lifetime income of their children and, hence, the marginal utility gain associated to this contribution does not depend on the tax rate. Therefore, the capital income tax reduces the bequest per capita left by individuals to their offspring. Moreover, this tax reduces the fraction of educated population since individuals require a larger amount of inheritance to invest in the education of their children.

Finally, we also obtain from (3.3) that the marginal increase in the rate of the capital income tax has an ambiguous impact on the individual's saving at the steady state. Since the capital income tax reduces the after-tax returns on saving, this tax displays an income effect and a substitution effect on saving. In other words, an increase in the tax rate stimulates saving to compensate the reduction in the disposable income at the old age, whereas the tax raises the amount of consumption at the adult age that the individual must sacrifice to obtain a unit of consumption at old age.

#### 5.4. Education subsidies: private vs. public education

In this subsection, we analyze the marginal effects of a subsidy to the education investment. For that purpose, we assume a marginal variation in the subsidy rate  $s_e$ ,

which is accommodated with public consumption. Obviously, this policy can affect only the bequest left by those individuals who invest in the education of their children since only those individuals are entitled to enjoy the subsidy. In particular, an increase in the subsidy rate raises the disposable income of these individuals and does not alter their contribution to the lifetime income of their children. Hence, the subsidy stimulates the willingness of parents to leave bequests, which reduces the level of inheritance that they must receive in order to invest in the education of their children. This conclusion can directly be proved by using conditions (3.2) and (3.9). Observe that a permanent increase in the subsidy rate raises the stationary fraction of population that is educated ( $h^i = 1 + \varepsilon$ ) and leave bequest equal to  $\bar{b}^2$  at the steady state. Moreover, the level of bequest  $\bar{b}^2$  left by each educated individual goes up, whereas the level of bequests  $\bar{b}^1$  left by non educated (i.e., the poorest) individuals does not change. Therefore, the aggregate income of adult individuals rises at the steady state and, moreover, the proportion of aggregate income enjoyed by poorest adult individuals decreases. Figure 4 illustrates the change in the relative frequencies of the stationary distribution of bequests after an increase in the subsidy rate on education.

[Insert Figure 4]

At this point one should ask whether the government can also affect the stationary distribution by providing directly part of the education. In other words, we can move the discussion to the differences between a private system of education and a (partially) public system of education in terms of intergenerational mobility and income inequality. For this purpose, we must analyze the effects of a marginal change in the value of the parameter  $\mu$  measuring the education cost faced by individuals. A change in the value of the cost  $\mu$  modifies accordingly the amount of government consumption since a fraction of education will be provided by the government and individuals will not have to pay directly for it. First, note from (3.4) that the functions  $B^1(b_t^i)$  and  $B^3(b_t^i)$  do not depend on  $\mu$ , whereas  $B^2(b_t^i)$  and  $B^4(b_t^i)$  depend negatively on this parameter. We then conclude that a marginal decrease in  $\mu$  raises the stationary level  $\bar{b}^2$  of bequest, whereas this marginal permanent shock in  $\mu$  does not alter the level  $\bar{b}^1$ . Thus, as was expected, the gap between  $\bar{b}^1$  and  $\bar{b}^2$  goes up when the cost of education faced by individuals decreases. Second, we obtain from (3.9) that a marginal decrease in  $\mu$  lowers the threshold level  $\tilde{b}$ . Hence, a decrease in the cost of education increase the number of initially non-educated dynasties converging to the steady state given by  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$ . Figure 5 shows the effects of an increase in the value  $\mu$  of the education cost on the stationary distribution of bequests.

[Insert Figure 5]

The previous results derive from the fact that a larger cost of education faced by individuals reduces the benefit that the parents obtain from leaving bequest to their children, which can be derived from comparing the two sides of equation (3.2). On the one hand, a larger education cost reduces the present value of the lifetime income that individuals devote to own consumption given the level of bequest left to their children. On the other hand, given the choice of parents with respect bequests and education investment, marginal changes in the education cost does not alter their contribution to

the lifetime income of their children. Therefore, the marginal effects of education cost on the level of bequest and the willingness to invest in education directly follows.

Our previous discussion shows that under a private system of education the initial aggregate income is smaller and the proportion of initial aggregate income enjoyed by the poorest adult individuals is larger at the steady state than under a partially public system. Furthermore, from (3.3) we obtain that the stationary saving per capita of the educated individuals is smaller in the private system, whereas the stationary saving per capita of the non-educated individuals does not depend on the education system. Thus, the aggregate saving at the steady state is larger when education is partially publicly provided than when it is entirely private.

Before closing this subsection, we should note that a reduction in the cost of education  $\mu$  has an impact qualitatively similar to the one derived from an increase in the education premium  $\varepsilon$ . These two parameters characterize the technology for human capital accumulation as they determine the productivity of the education system. This productivity is obviously an increasing function of the education premium  $\varepsilon$  and a decreasing function of the education cost  $\mu$ . Therefore, the more productive the education sector, the larger the mobility in the human capital, so that the larger the fraction of population that is educated and the larger the income inequality between educated and non-educated individuals.

### 5.5. Social security system

We now analyze the effects of a pay-as-you-go social security system where old individuals receive lump-sum benefits that are financed by the contributions of adult individuals. If the contributions of adult individuals take the form of labor income taxes, the social security system has two distortionary effects on our economy. On the one hand, since the labor supply in efficiency units is endogenous, the labor income tax distorts the individual choice as was showed in Subsection 5.2. On the other hand, the social security system implies an ex-ante intergenerational redistribution from adult to old individuals. Since we have already studied the distortionary effects of labor income taxation, we will exclusively focus on the intergenerational redistribution effects achieved through lump-sum taxes. Therefore, we now assume that  $\tau_w = \tau_b = \tau_k = s_e = 0$  and analyze the intergenerational redistribution effect by considering a marginal variation of the lump-sum tax  $\lambda$  satisfying

$$\frac{d\theta}{d\lambda} = -n \quad (5.8)$$

in order to fulfil the budget constraint (2.1). Therefore, we will assume that the government increases the lump-sum tax  $\lambda$  paid by the adult individuals, and the additional revenues are devoted to finance an increase in the lump-sum subsidy  $\theta$  to the old individuals.

The effects of a pay-as-you-go social security system on income distribution and on the intergenerational mobility of human capital depend on whether the economy is dynamically efficient or inefficiency as defined by Cass (1979). In particular, an increase in the lump-sum transfer from adult to old individuals (an increase in  $\lambda$ ) has qualitatively the same stationary effects as a rise in the capital income tax if  $1+r > n$ , whereas this variation on the intergenerational transfer has qualitatively the opposite

effects to a rise in the capital income tax when  $1+r < n$ . This is true except for the relative effect on the stationary level of bequests  $\bar{b}^1$  and  $\bar{b}^2$ . Variations in the social security transfers never alter the gap between these two stationary levels of bequests. The previous conclusions are easily derived by differentiating (3.4) and (3.9) with respect to  $\lambda$ . Using (5.8), we obtain

$$\frac{\partial B(b_t^i, h_t^i, e_t^i)}{\partial \lambda} = \frac{\beta(1+r) \left( \frac{n}{1+r} - 1 \right)}{n(1+\beta+\rho)},$$

and

$$\frac{\partial \tilde{b}}{\partial \lambda} = \left( 1 - \frac{n}{1+r} \right).$$

Following then the same procedure used for the other fiscal instruments, we directly derive the previous characterization of the effects of the social security system.

The social security program distorts the decisions on bequest and education investment by affecting the marginal utility loss derived from reducing consumption in order to increase the amount of bequest left to children. In particular, there may exist a wedge between the returns on saving given by  $R(\tau_k)$  and the returns of social security program given by  $n$ . Thus, this policy can alter the present value to the lifetime income that individuals devote to own consumption given the level of bequest that they will leave to their children. If  $R(\tau_k) > n$ , then the social security program reduces the present value of lifetime income of individuals. Therefore, in this case this policy reduces the willingness of parents to leave bequest and to invest in the education of their children. Evidently, the opposite conclusion is derived when  $R(\tau_k) < n$ .

## 6. Conclusion

This paper has analyzed how the investment in human capital determines both the intragenerational income distribution and the intergenerational mobility in human capital. The analysis shows that not only the initial distribution of wealth, but also the distribution of the composition of wealth between bequests and human capital, are important to characterize the evolution of both distribution of income and intergenerational mobility in human capital. There are three main assumptions that give rise to our result. First, the education of individuals can only be financed by their parents, who derive satisfaction from their contribution to the lifetime income of their children with independence of the type of the intergenerational transfers used for that purpose. Second, we assume that the acquisition of human capital is indivisible and requires thus a minimum amount of investment. Finally, individuals do not have access to the credit market so that those individuals with a sufficiently small income can not afford the investment in the education of their children. Hence, intragenerational income distribution and intergenerational mobility in human capital are affected by the percentage of individuals who inherited a sufficiently large level of physical wealth to enable them to invest in the education of their offspring. Furthermore, the minimum level of inheritance required by educated parents to give education to their children differs from the minimum level for non-educated parents.

From our results we can conclude that the regional differences in the composition of intergenerational transfers, income inequality, and mobility are determined by: (i) differences in the initial distribution of wealth; (ii) differences in the initial distribution of the composition of wealth; (iii) differences in the fiscal policy set by the governments; (iv) differences in the degree of imperfection of the credit market; and (v) differences in the process of human capital accumulation. Regarding to the latter determinant, the first candidate for generating regional differences is the education technology. In particular, the regions can differ in the education costs, in the number and length of education levels, or in the productivity of the technology used to accumulate human capital. Moreover, the regional differences in the process of human capital accumulation can also arise from the amount of public resources invested in education. In this respect, we have proved that the distribution of income and the mobility under a private system of education differ from those under a public system.

A natural and promising extension of our research is to analyze the implications that our results have for economic growth and development. This requires a generalization of the process of human capital accumulation in order to allow for some intergenerational transmission of embodied human capital. Moreover, the assumption of constant interest and wage rates should be modified accordingly along the lines of Owen and Weil (1998). The analysis would face then the challenge of dealing with an evolution within each dynasty that will depend on the aggregate income distribution.

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## Appendix

### A. Optimality conditions of the individual problem

We derive in this appendix the optimal conditions on  $c_t^i$ ,  $x_t^i$ ,  $s_t^i$  and  $b_{t+1}^i$ . To this end we take the value of  $e_t^i$  as given. First, by combining (2.5) and (2.6) we obtain the following intertemporal budget constraint:

$$(1 - \tau_w)w_t h_t^i + (1 - \tau_b)b_t^i - \Omega = c_t^i + (1 - s_e)ne_t^i + \frac{x_{t+1}^i + nb_{t+1}^i}{R(\tau_k)}. \quad (\text{A.1})$$

Second, consider the problem consisting on maximizing (2.2) with respect to  $\{c_t^i, x_{t+1}^i, b_{t+1}^i\}$  subject to (A.1) and (2.7). Denote by  $\psi$  the Lagrangian multiplier associated with the constraint (A.1). The first order conditions of the previous problem are given by

$$c_t^i = \frac{1}{\psi}, \quad (\text{A.2})$$

$$x_{t+1}^i = \frac{\rho R(\tau_k)}{\psi}, \quad (\text{A.3})$$

and

$$\frac{\beta(1 - \tau_b)}{\bar{I}_{t+1}^i} \leq \frac{n\psi}{R(\tau_k)}. \quad (\text{A.4})$$

By combining (A.2) and (A.3), we directly get (3.1). Moreover, from (A.2) and (A.3) we obtain

$$c_t^i + \frac{x_{t+1}^i}{R(\tau_k)} = \psi(1 + \rho). \quad (\text{A.5})$$

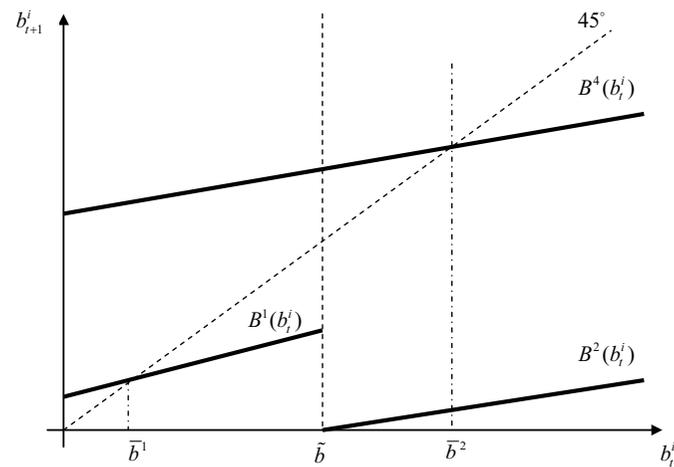
Combining (A.1), (A.4), and (A.5) we can easily derive equation (3.2). Finally, after solving for  $\psi$  in condition (A.4) when it holds with equality and substituting the result in (A.2) and (A.3), we obtain conditions (3.5) and (3.6).

### B. Different configurations of the stationary distribution

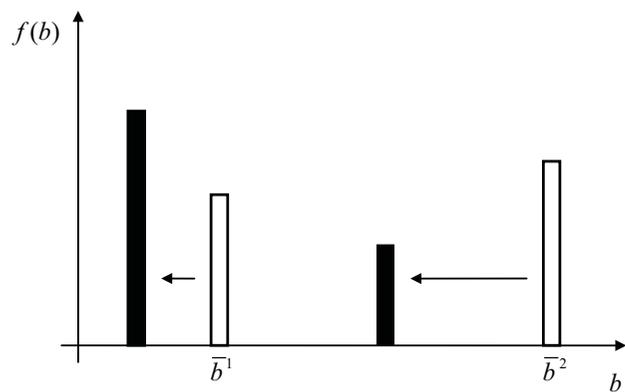
The conditions (4.4), (4.5), (4.6) and (4.7) determine the configuration of the stationary distribution of bequests and human capital. In particular, the following configurations of the stationary distribution can emerge in our theoretical economy:

1. When condition (4.4) does not hold, then no stable stationary distribution exists.
2. When condition (4.4) holds, the economy converges to a degenerate distribution if at most one of the following situations occurs: (i) condition (4.6) does not hold; or (ii) at least one of the conditions (4.5) and (4.7) is not satisfied. First, if condition (4.6) does not hold, then the fixed point  $\bar{b}^1$  is the unique interior steady-state. In this situation all dynasties leave an amount of bequest equal to  $\bar{b}^1$  and do not invest in the education of their children. Second, if at least one of the conditions (4.5) and (4.7) does not hold, then the fixed point  $\bar{b}^2$  is the unique interior steady-state. In this case all the dynasties invest in their children's education and leave an amount of bequest equal to  $\bar{b}^2$ .

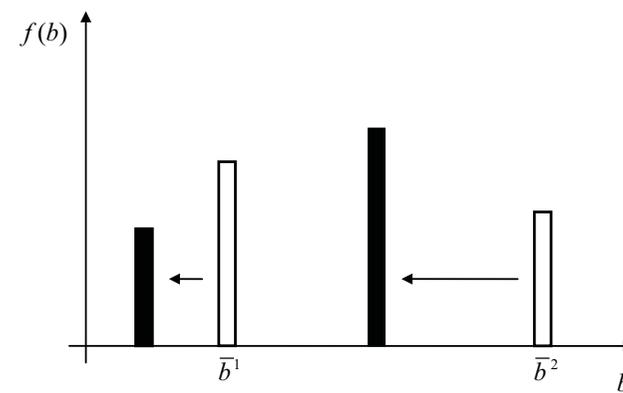
3. When condition (4.4) holds, and condition (4.6) together with at least one of conditions (4.5) and (4.7) does not, then no stationary distribution exists.
4. Finally, when all the conditions (4.4), (4.5), (4.6) and (4.7) hold, then the economy converges to a two-point distribution where some dynasties leave a bequest per children equal to  $\bar{b}^1$  and do not invest in their education, whereas other dynasties do invest in the education of their children and leave a bequest per capita equal to  $\bar{b}^2$ .



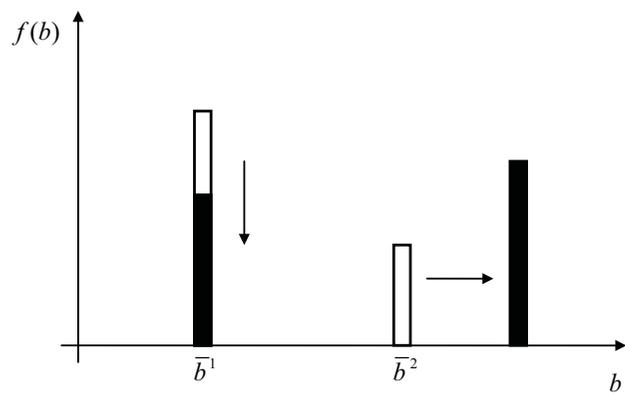
**Figure 1.** The dynamics of bequests.



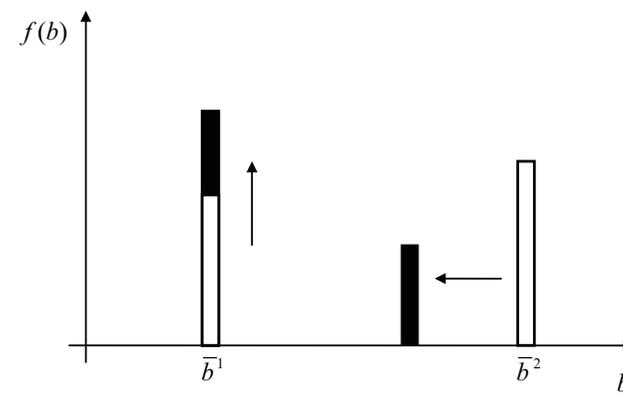
**Figure 2.** The effect on the distribution of bequests of a rise in the inheritance tax, in the capital income tax, or in the labor income tax when  $\varepsilon < \varepsilon^*$ .



**Figure 3.** The effect of a rise in the labor income tax on the distribution of bequests when  $\varepsilon > \varepsilon^*$ .



**Figure 4.** The effect of a rise in the education subsidy on the education subsidy rate.



**Figure 5.** The effect on the distribution of bequests of a rise in the education cost.