Engels’ Pause:

A Pessimist’s Guide to the British Industrial Revolution

by

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Abstract

The paper reviews the macroeconomic data describing the British economy from 1760 to 1913 and shows that it passed through a two stage evolution of inequality. In the first half of the nineteenth century, the real wage stagnated while output per worker expanded. The profit rate doubled and the share of profits in national income expanded at the expense of labour and land. After the middle of the nineteenth century, real wages began to grow in line with productivity, and the profit rate and factor shares stabilized. An integrated model of growth and distribution is developed to explain these trends. The model includes an aggregate production function that explains the distribution of income, while a savings function in which savings depended on property income governs accumulation. Simulations with the model show that technical progress was the prime mover behind the industrial revolution. Capital accumulation was a necessary complement. The surge in inequality was intrinsic to the growth process: Technical change increased the demand for capital and raised the profit rate and capital’s share. The rise in profits, in turn, sustained the industrial revolution by financing the necessary capital accumulation. After the middle of the nineteenth century, accumulation had caught up with the requirements of technology and wages rose in line with productivity.

Key words: British industrial revolution, kuznets curve, inequality, savings, investment

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“Since the Reform Act of 1832 the most important social issue in England has been the condition of the working classes, who form the vast majority of the English people...What is to become of these propertyless millions who own nothing and consume today what they earned yesterday?...The English middle classes prefer to ignore the distress of the workers and this is particularly true of the industrialists, who grow rich on the misery of the mass of wage earners.”


Pessimism has been the usual response to the Industrial Revolution. Malthus, Ricardo, and Marx set the tone, for the classical economists all believed that real wages remained constant even though capital accumulation and technical progress were boosting output per worker. Many social commentators and foreign travellers agreed (O’Brien and Riello 2004). A variety of economic and social indicators also support this view. For instance, the per capita consumption of working class luxuries like sugar and tea was flat during the industrial revolution (Mokyr 1988), per capita food production and calorie consumption per head declined between 1750 and 1850 (Allen 1999, 2005), woman lost income earning assets (Humphries 1990, Horrell and Humphries 1995), the expectation of life at birth fell in large cities in the second quarter of the nineteenth century (Szreter and Mooney 1998), and the heights of a variety of low occupation groups dropped over the course of the industrial revolution (Floud, Wachter, and Gregory 1990, Johnson and Nicholas 1995, Nicholas and Oxley 1993, Nicholas and Steckel 1991 Komlos 1993).

Pessimism has been much more controversial, however, among economists who have measured the trend in real wages. Lindert and Williamson (1983) were the first to apply modern economic methods to the question. They computed economy-wide average earnings and a consumer price index founded on budget surveys and corresponding prices. Their conclusion was guardedly ‘optimistic’ in that they found the average real wage rose sharply after 1815. This conclusion was not universally accepted. Feinstein (1998) computed an alternative price index that significantly reduced the rate of real wage growth leading to his title ‘pessimism perpetuated.’ Clark (2005), on the other hand, proposed yet another price index that tilted the conclusions back in a Lindert-Williamson direction. Recently, Allen (2007) has reviewed the procedures of Feinstein and Clark to construct the best possible index with currently available information. While Clark has introduced better measures of some prices, Allen’s new index tells a story very similar to Feinstein’s–thus, pessimism is preserved. This article adopts Allen’s new real index as the measure of working class incomes. Redoing the analysis with Feinstein’s index gives almost identical results. Indeed, despite Clark’s optimism, his index shows Engel’s pause in real wage growth just as clearly as Feinstein’s and Allen’s, although factor shares move less dramatically against labour with Clark’s index than with the others. While the main text is based on Allen’s index, Appendix II contains some of the corresponding graphs based on Clark’s index. The essential conclusions remain.

Why did the real wage grow slowly during the industrial revolution? Lewis (1954) offered one solution to this problem with his famous model of ‘economic development with unlimited supplies of labour.’ Conceptually, he divided the economy into two sectors: one was peasant agriculture where the population was in surplus, capital was scarce, the marginal product of labour was zero, and income sharing guaranteed subsistence to all. The other was
the modern, industrial sector where capital intensive production meant that labour productivity was high. Growth occurred as the modern sector expanded through capital accumulation. Labour to man the new capacity was available from the agricultural sector in infinitely elastic supply at the subsistence wage. This supply condition kept wages in the modern sector at subsistence—pessimism in action!—with the result that profits increased. The increase in profits provided the savings that allowed the modern sector to enlarge. When it was large enough to absorb all the labour surplus, further accumulation meant that wages rose along with productivity. The result was a two stage growth process with rising inequality in the first stage followed by a more equitable growth trajectory in the second.

Britain is seen to illustrate Lewis’ two stage process if we combine Crafts’ (1985a) and Feinstein’s (1972) GDP estimates with any of the real wage series. Figure 1 illustrates the pattern with Allen’s series (although it must be kept in mind that the pre-1860 GDP series is interpolated between a few benchmark estimates\(^1\)). The first stage lasted from the eighteenth century until the 1840s. From about 1790 to about 1840—the period of Engel’s pause—the real wage scarcely budged, while output per head rose 37%. That increase in GDP accrued to someone—and it wasn’t workers. The second phase lasted from the 1840s into the twentieth century. In this period, the real wage increased in line with the growth in output per worker.

While Figure 1 is consistent with Lewis’ schema, the mechanisms he outlines do not explain it. As a general matter, the Lewis model has presented growth theorists with many difficult problems. In addition, there are particular problems to applying it to the British industrial revolution. Agriculture did not function as source of surplus labour that kept wages down. For one thing it was too small. In 1800 only about 35% of the English work force was in agriculture (Allen 2000, p. 8) compared to the 75 - 80% that characterized the less developed countries Lewis was describing. Moreover, contrary to Marx, the parliament enclosures did not drive workers from the land; indeed, the poor law (through the Speenhamland system) paid men to stay in the countryside and reduced rural-urban migration. Finally, while real wages were stagnant in industrializing Britain, they stagnated at a high level when seen internationally: real wages in eighteenth century Britain and the Low Countries were higher than anywhere else in Eurasia (Allen 2001). This does not square with Lewis’ scenario.

We can avoid the implausibilities of the Lewis model with an integrated model of growth and distribution. This model is a Solow (1956) one sector growth model in which savings are a function of property income rather than total income—here we do incorporate one of Lewis’s themes. Suitably calibrated, the model closely tracks the growth and inequality history of the industrial revolution. Simulations from 1760 to 1913 reproduce the two phases of rising and then constant inequality that Lewis delineated. Finally, the model allows us to probe the causes of inequality more deeply. While the classical economists all expected the real wage to remain constant, they disagreed about the reason: Malthus and Ricardo emphasized the growth of population, while Marx emphasized the labour saving bias of technical change. We can establish the importance of these explanations by simulating the integrated model.

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\(^1\)Broadberry, Földvári, and van Leeuwen (2006) have presented preliminary estimates of real GDP annually. These are ‘work in progress’ but show the same trends as the series used here.
The macro-economic record

Our knowledge of the macroeconomics of the industrial revolution is fuller than it was fifty years ago thanks to the work of Deane and Cole (1969), Wrigley and Schofield (1981), McCloskey (1981), Crafts (1976, 1983, 1985), Harley (1982, 1993), and Crafts and Harley (1992), and, most recently, Antràs and Voth (2003). We now have well researched estimates of the growth of real output, the three main inputs (land, labour, and capital), and overall productivity. More recent research by Feinstein (1998), Allen (1992, 2007), Turner, Beckett, and Afton (1997), and Clark (2002, 2005) has filled in our knowledge of input prices—real wages and land rents.

Most research has aimed to measure GDP and the aggregate inputs. A major finding is that the rate of economic growth was slow but still significant. Indeed, each revision of the indices of industrial output or GDP from Hoffmann (1955) to Deane and Cole (1969) to Harley (1982) to Crafts (1985) to Crafts and Harley (1992) has seen a reduction in the measured rate of economic growth. Likewise, real wage growth has decelerated from Lindert and Williamson (1983) to Crafts (1985) to Feinstein (1998). The latest estimates of GDP growth, on a per head basis, are shown in Table 1. Between 1761 and 1860, output per worker in Great Britain rose by 0.6% per year. Growth was slower before 1801 and accelerated thereafter. Even the fastest growth achieved (1.12% per year) was very slow by the standards of recent growth miracles where rates of 8% or 10% per year have been realized. Nevertheless, between 1760 and 1860, per capita output increased by 82%. This was an important advance in the history of the world.

Standard theories of economic development have been used to explain this increase. Lewis’ theory attributed economic expansion to a rise in the investment rate:

The central problem in the theory of economic development is to understand the process by which a community which was previously saving and investing 4 or 5 per cent of its national income or less converts itself into an economy where voluntary saving is running at about 12 to 15 per cent of national income or more. This is the central problem because the central fact of economic development is rapid capital accumulation. (Lewis 1954.)

Testing this theory required Feinstein’s (1978, p. 91) estimates of investment and Craft’s (1985, p. 73) estimates of GDP. Dividing one by the other showed that gross investment rose from 6% of GDP in 1760 to 12% in 1840. Although the pace of the increase was modest, the change in the British investment rate during the industrial revolution was consistent with the Lewis model.

The analysis of growth was transformed with the publication of Solow’s (1957) justification of growth account and his application of the methodology to twentieth century America, for which he claimed that technical progress, rather than capital accumulation, was the main cause of growth. To see whether industrializing Britain was the same, measures of the capital stock, labour force, and land input were needed as well as a series of real GDP.

Table 1 shows Solow’s growth accounting model applied to Britain. In this approach, some of the rise in output per capita is attributed to the increase in the capital-labour ratio and some to the growth (in this case decline) in the ratio of land to labour. The second factor was negative and the first slight. The growth in total factor productivity more than equalled the growth in per capita output in 1800-30 and 84% of its growth in 1830-60. The overall conclusion is that the growth in income per head was almost entirely the result of
technological progress—just like twentieth century America. Further research indicates that productivity growth in the famous, ‘revolutionized’ industries and in agriculture was enough to account for all of the productivity growth at the aggregate level (Crafts 1985a, Harley 1993). Productivity growth was negligible in other sectors of the economy. Lewis’ emphasis on capital accumulation and growth accounting’s emphasis on technical progress have not been formally integrated, at least in the context of the industrial revolution. The rise in the investment rate was, presumably, an endogenous response to the possibilities of making money from the new technologies of cotton, iron, and steam. But was the stimulus really large enough to provoke the response? We need a model with endogenous capital accumulation to find out.

The macro-economic record of inequality

While there is considerable consensus about the evolution of aggregate inputs and outputs, there is less agreement about the history of inequality. Much research has been guided by Kuznets’ (1955) conjecture that inequality rises during early industrialization and declines as the economy matures (although it has risen again in the last few decades). The most direct approach to testing that would compare inequality indices like Gini coefficients at different points in the industrialization process. Lindert and Williamson (1983b) and Williamson (1985) tried this, but the comparability of the data has been questioned (Feinstein 1988a). I concentrate on the functional distribution of income, which, in any event, is easier to address with simple models. Ownership of land and capital were concentrated in industrializing Britain, so a rise in property income signals an increase in inequality. This focus is also appropriate if one approaches inequality from the perspective of Victorian debates, which emphasized distribution between social classes defined by ownership of factors of production: In the Ricardian analysis of the corn laws, for instance, the gains from growth accrued to landlords, while in the writing of Marx and Engels the free enterprise system directed income from workers to capitalists.

Factor prices and factor shares are graphed in Figures 1-3. All values are real returns and real shares measured in the prices of the 1850s. I consider them in the order in which they were constructed.

The average real wage shows an upward trend from 1770 to 1800, then a plateau until about 1840, when the index resumes its ascent. The eighteenth century rise contributed to stable inequality, while the early Victorian plateau contributed to rising inequality.

The real rent of land rose slowly over the century from 1760 to 1860 (Clark 2002, p. 303). Pace Ricardo, it does not play a major role in the surges of inequality.

By multiplying the real wage by the occupied population and the real rent by the cultivated land, one obtains the wage bill and total rent. Dividing these by real GDP gives the shares of labour and land. Subtracting these from one gives capital’s share. Capital income (“profits”) in this context includes the return to residential land, mines, entrepreneurship, and the premium of middle class salaries and self-employment income over the wages of manual labor.

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2This view has been disputed by Berg and Hudson (1992) and Temin (1997).
workers.\textsuperscript{3} Capital income as used here is closer to Marxian surplus value than to profits or interest narrowly defined.

The shares are graphed in Figure 2. The share of rent in national income declined gradually over the century. The shares of wages and profits exhibited conflicting trends. Labour’s share rose very slightly between 1770 and 1800 and then declined significantly until 1840 when the situation stabilized. Capital’s share moved inversely, falling in the 1780s and 1790s and then surging upward between 1800 and 1840. Capitalists gained at the expense of both landlords and labourers. The former advance may not have increased inequality, but the latter certainly did.

Finally, one can calculate the gross profit rate from equation 6 by multiply capital’s share by real GDP and then dividing the product by Feinstein’s estimate of the real capital stock (Figure 3). Also shown are analogous profit rates computed from Deane and Cole (1969, pp. 166-7) for 1801 onwards.\textsuperscript{4} The gross profit rate was low and flat in the eighteenth century and jumped upwards between 1800 and 1840. Interest rates do not show the same increase, but they are a narrower measure of profits than that used here. Moreover, interest rates were too heavily regulated to be a reliable indicator of the demand for capital. Temin and Voth (2005) found that Hoare’s bank rationed credit instead of raising interest rates.

The case for rising inequality in the first half of the nineteenth century, therefore, rests on the stagnant real wage, the decline in labour’s share of the national income and the rise in capital’s, and the increase in the gross profit rate.

A Model of Growth and Income Distribution

While we have a clearer understanding of the trends in the British economy between 1760 and 1860, there are still important questions about the economic processes that governed its evolution. How were technical progress and capital accumulation interconnected? What determined the rate of investment? Why did inequality increase after 1800 and then stabilize from the middle of the nineteenth century to the First World War? Was the rise in inequality an incidental feature of the period or a fundamental aspect of the growth process? To answer these questions, we need a model of the economy. The model proposed here is of the simplest sort. Only one good (GDP) is produced, and it is either consumed or invested. Agriculture and manufacturing are not distinguished. As a result, issues like the inter-sectoral terms of trade are not modelled. Important social processes like urbanization lurk in the aggregates, however, as the population grows and capital is accumulated without much increase in cultivated land.

\textsuperscript{3}As explained in the Appendix, the real wage index is normalized to equal Deane and Cole’s (1969, pp. 148-53) estimate of average earnings in 1851. Their earnings estimate is based on the wages of manual workers and so excludes the higher salaries of the middle class.

\textsuperscript{4}They estimated nominal property income, mainly from tax sources. Subtracting nominal agricultural rent gives an estimate of nominal profits at ten year intervals. Dividing each year’s profits by the value of the capital stock series (recalculated in the prices of that year) gives the historical profit rates shown in Figure 3.
The model proposed here, however, does integrate growth and income distribution. Output and factor prices are determined by neoclassical production function and its marginal products. Savings depends on property income and, thus, on the distribution of income, which, therefore, also feeds back on the growth rate. I begin with the three equations that comprise the heart of the Solow (1956) growth model:

\[ Y = f(AL, K, T) \]  
\[ K_t = K_{t-1} + I_t - \delta K_{t-1} \]  
\[ I = sY \]

The first is a neoclassical production function in which GDP \( Y \) depends on the aggregate workforce \( L \), capital stock \( K \), and land area \( T \). The latter is not normally included in a Solow model but is added here due to its importance in the British economy during the Industrial Revolution. \( A \) is an index of labour augmenting technical change. Technical change of this sort is necessary for a continuous rise in per capita income and the real wage.

The second equation defines the evolution of the capital stock. The stock in one year equals the stock in the previous year plus gross investment \( I \) and minus depreciation (at the rate \( \delta \)) of the previous year’s capital stock.

The third equation is the savings or investment function according to which investment is a constant fraction \( s \) of national income. Equation (3) is the very simple Keynesian specification that Solow used. In some simulations, I will use it to set the economy-wide savings rate. However, equation (3) is not descriptive of industrializing Britain where all saving was done by landlords and capitalists. This idea is incorporated into the model with a savings function along the lines of Kalecki (1942) and Kaldor (1956):

\[ I = (s_K \phi_K + s_r \phi_r)Y \]  

In this specification, capitalists and landowners do all the savings since \( s_K \) is the propensity to saving out of profits and \( \phi_K \) is the share of profits in national income. Likewise, \( s_r \) is the propensity to saving out of rents and \( \phi_r \) is the share of rents. The economy-wide savings rate \( s = (s_K \phi_K + s_r \phi_r) \) depends on the distribution of income. With equation 4, accumulation and income distribution are interdependent and cannot be analysed separately. In other words, one cannot first ask why income grew and then ask how the benefits of growth were distributed. Each process influenced the other.

Usually, a growth model also includes an equation specifying the growth in the workforce or population (assumed to be proportional) at some exogenous rate. Since the model is

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\(^5\)David (1978) analysed American growth with a model like this and called it a “Cantabridgian Synthesis” since it incorporated elements of both the Cambridge, Massachusetts, and Cambridge, England, styles of growth models. Samuelson and Modigliani (1966) analysed the model theoretically and called it “a Neoclassical Kaldorian Case” (p. 295). They anticipated the Cantabrigian terminology with their quip that their analysis “can encompass valid theories in Cambridge, Massachusetts, Cambridge, Wisconsin, or any other Cambridge.” (P. 297).
Van Zanden (2005) uses a Solow model with a Cobb-Douglas function to analyze early modern economic growth. Introduced by Christensen, Jorgenson, and Lau (1971) and Layard, Sargan, Ager, and Jones (1971).

Three more equations model the distribution of income explicitly. The derivatives of equation (1) with respect to L, K, and T are the marginal products of labour, capital, and land, and imply the trajectories of the real wage, return to capital, and rent of land. These factor prices can also be expressed as proportions of the average products of the inputs:

\[
\begin{align*}
    w &= \phi_L \frac{Y}{L} \\
    i &= \phi_K \frac{Y}{K} \\
    r &= \phi_T \frac{Y}{T}
\end{align*}
\]

Here \(w\), \(i\), and \(r\) are the real wage, profit rate, and rent of land. \(\phi_L\), \(\phi_K\), \(\phi_T\) are the shares of labour, capital, and land in national income, as previously noted.

A production function must be specified to apply the model to historical data. The Cobb-Douglas is commonly used, and, indeed, I used a Cobb-Douglas for trial simulations and to determine a provisional trajectory for productivity growth. The function is:

\[
Y = A_0 (AL)^{\alpha} K^\beta T^\gamma
\]

where \(\alpha\), \(\beta\), \(\gamma\) are positive fractions that sum to one when there is constant returns to scale, as will be assumed. \(A_0\) is a scaling parameter. With a Cobb-Douglas technology, \(A\) can be factored out as \(A^a\) which is the conventional, Hicks neutral, total factory productivity index. In addition, in competitive equilibrium, the exponents \(\alpha\), \(\beta\), and \(\gamma\) equal the shares of national income accruing to the factors (\(\phi_L\), \(\phi_K\), and \(\phi_T\)). These shares are constants. They can be calculated from the national accounts of one year; in other words, the model can be calibrated from a single data point.\(^6\)

Ultimately, however, the Cobb-Douglas is not satisfactory for understanding inequality since the essence of the matter is that the shares were not constant. Economists have proposed more general functions that relax that restriction. The simplest is the CES (constant elasticity of substitution). It is not general enough, however, for it requires that the elasticities of substitution between all pairs of inputs be equal (although not necessarily equal to one). Instead, I have used the translog production function.\(^7\) It is the natural generalization of the Cobb-Douglas. With the translog, all shares can vary as can all of the pair-wise elasticities of substitution. The translog is usually written in logarithmic form:

\[^6\text{Van Zanden (2005) uses a Solow model with a Cobb-Douglas function to analyze early modern economic growth.}\]

\[^7\text{Introduced by Christensen, Jorgenson, and Lau (1971) and Layard, Sargan, Ager, and Jones (1971).}\]
\[ \ln Y = \alpha_0 + \alpha_k \ln K + \alpha_l \ln (AL) + \alpha_t \ln T + \]
\[ \frac{1}{2} \beta_{kk} (\ln K)^2 + \beta_{kl} \ln K \ln (AL) + \beta_{kt} \ln K \ln T + \]
\[ \frac{1}{2} \beta_{ll} (\ln (AL))^2 + \beta_{lt} \ln (AL) \ln T + \frac{1}{2} \beta_{tt} (\ln T)^2 \quad (9) \]

subject to the adding up conditions \( \alpha_k + \alpha_l + \alpha_t = 1, \beta_{kk} + \beta_{ll} + \beta_{tt} = 0, \beta_{kl} + \beta_{lt} + \beta_{kt} = 0, \)
and \( \beta_{kk} + \beta_{ll} + \beta_{tt} = 0. \) When all of the \( \beta_{ij} = 0, \) the translog function reduces to the Cobb-Douglas.

Logarithmic differentiation of the translog function gives share equations that imply trajectories of factor prices in accord with equations 5-7:

\[ s_K = \alpha_k + \beta_{kk} \ln K + \beta_{kl} \ln (AL) + \beta_{kt} \ln T \quad (10) \]
\[ s_L = \alpha_l + \beta_{lk} \ln K + \beta_{ll} \ln (AL) + \beta_{lt} \ln T \quad (11) \]
\[ s_T = \alpha_t + \beta_{tk} \ln K + \beta_{tl} \ln (AL) + \beta_{tt} \ln T \quad (12) \]

These equations are the basis for calibrating the model, as we will see.

**Savings and Production Function Calibration**

The savings and production functions are central to the growth model, and each must be estimated. Were there sufficient data, this could be done econometrically, but data are too limited for that. Instead they are calibrated.

There are two variants of the savings function. In the case of \( I = sY \) (equation 2), \( s \) is determined by dividing real gross investment by real GDP. The ratio rises gradually from about 6% in 1760 to 11% in the 1830s and 1840s. It sags to about 10% in the 1850s.

The alternative savings function is the Kalecki function \( I = (s_K K + s_T T)Y \) (equation 4). This function is preferred for two reasons. First, household budgets from the industrial revolution indicate that, on average, workers did not save. In some cases, income exceeded expenditure by a small amount; in others, the reverse was true. Overall, there was no net savings (Horrell and Humphries 1992, Horrell 1996). All of the savings, therefore, came from landlords and capitalists. Figure 4 shows the ratio of savings to their income. There is some suggestion that the savings rate out of property income rose in the 1760s and 1770s, but thereafter there was no trend. Regression of the savings rate on the shares of profits and rents in national income for the period 1770-1913 showed a small difference between landlords and capitalists:

\[ I/Y = .138 \phi_T + .196 \phi_K \quad (13) \]

The coefficients had estimated standards errors of .013 and .004 respectively. In this model, capitalists saved a higher proportion of income than landlords. I used this equation for most simulations except that I lowered the coefficient of savings by capitalists to .14 in the 1760s and .16 in the 1770s. This improved the simulations in those years and creates a small exogenous component to the rise in savings from 6.5% in 1760 to 7% in 1780. The increase in savings in later years remains dependent on changes in the distribution of income.
The parameters of the translog function must also be determined. While the parameters of the Cobb-Douglas function can be calculated from the factor shares at one point in time, the translog requires two sets of factor shares. If the adding up conditions $\alpha_k + \alpha_l + \alpha_t = 1$, $\beta_{kl} + \beta_{ll} + \beta_{tl} = 0$, and $\beta_{kt} + \beta_{lt} + \beta_{tt} = 0$ are imposed on equations 10-12, one gets:

\[
\begin{bmatrix}
  s_k & 1 & 0 & \ln K & \ln L & \ln T & 0 \\
  s_l & 0 & 1 & 0 & \ln K-\ln AL & \ln AL-\ln T & -\ln AL-\ln T \\
  s_t - 1 & -1 & -1 & 0 & 0 & \ln K-\ln AL & \ln T-\ln AL \\
\end{bmatrix}
\begin{bmatrix}
  \alpha_k \\
  \alpha_l \\
  \beta_{kk} \\
  \beta_{kl} \\
  \beta_{kt} \\
  \beta_{tt}
\end{bmatrix}
\]

If the values for the three shares and the corresponding $K$, $T$, $L$, and $A$ are substituted into these three equations for two years, then one obtains six equations in the six unknown parameters $\alpha_k$, $\alpha_l$, $\beta_{kk}$, $\beta_{kl}$, $\beta_{kt}$, and $\beta_{tt}$. These can be solved by inverting the matrix and premultiplying the share vector with it. The remaining parameters can be calculated from the imposed conditions.

After some experimentation, I used $\phi_k = .53$, $\phi_l = .28$, and $\phi_t = .19$ in 1770 and .47, .43, and .1 in 1860. Values for $A$ at these dates are also necessary. Indeed, the entire trajectory of $A$ from 1760 to 1860 is necessary for later simulations. Trial rates for the periods 1760-1800, 1801-30, and 1831-60 have been obtained both from the growth rates of residual productivity in Table 1 (using the relationship that residual TFP equals the rate of labour augmenting technical progress raised to the power of labour’s share) and by simulating a simplified version of the model with a Cobb-Douglas production function. The rates are refined by iterating between the translog production function parameters and the trajectory of labour augmenting technical change until a close fit between actual and simulated GDP is obtained. The estimated rate of labour augmenting technical change increased from .3% per year in 1760-1800 to 1.5% in 1801-30 and, finally, to 1.7% in 1831-60. No distributional information between 1770 and 1860 is used to calibrate the model; so its ability to replicate Engel’s pause (as we will see) is independent verification of the model rather than an artefact of its construction.

The estimated translog parameter values are shown in Table 2. Their magnitudes depend on the units in which the inputs are measured. For intelligibility, labour and land were rescaled to equal 248 in 1760, which was the value of the capital stock at the time. In view of the adding up restrictions $\beta_{kk} + \beta_{lk} + \beta_{tk} = 0$, $\beta_{kl} + \beta_{ll} + \beta_{tl} = 0$, and $\beta_{kt} + \beta_{lt} + \beta_{tt} = 0$ in equations 10 - 12, the implied shares for 1760 equal $\alpha_k$, $\alpha_l$, and $\alpha_t$, and the production function looks Cobb-Douglas. Differentiating equations 10 -12 with respect to time gives differential equations accounting for the change in shares in terms of the growth of the inputs. Because of technical progress, augmented labour grew more rapidly than any other. The
growth in augmented labour drove the evolution of the shares: Since $\beta_{LL}$ and $\beta_{LT}$ were negative, while $\beta_{LK}$ was positive, the shares of labour and land declined during the industrial revolution, while the share of capital increased. The economic significance of these arithmetic features is shown shortly when we work out their implications of the elasticity of substitution between capital and labour.

How well does the model perform?

To see how well the model performs, we need to simulate it with historical values for the exogenous variables to check that the simulated values of the endogenous variables track their historical counterparts.

First, does the model track GDP? Figure 5 compares the actual and simulated series and shows that they are almost indistinguishable.

Second, does the model track the investment rate? Figure 6 compares the two, and the trends are similar. The saw toothed pattern in the historical series reflects Feinstein’s presentation of his investment figures as ten year averages. The simulated investment series follows the upward trend of Feinstein’s series. Some of the rise 1760-80 is due to the exogenous increase in the propensity to save out of profits. Otherwise, the growth in the investment rate is due to the shift of income to capitalists.

Third, can the model explain the changes in factor shares? Figure 7 shows the simulation of labour’s share, which is of key importance. The simulation follows the trend of the actual series closely. Figure 8 compares historical and simulated values of all three shares. The predicted share of land matches the actual share closely, including the flat phase of the late eighteenth century and the halving of the share from 1800 to 1860. The simulation also does a good job tracking capital’s share.

Next consider factor prices. Figure 9 compares the historical real wage with the real wage series implied by equation 5 evaluated for the translog production function. The translog simulation mimics the upward trend in the late eighteenth century, the stagnation from 1800 to 1840, and the subsequent ascent.

Figure 10 contrasts the simulated trajectories of the gross profit rate (profits divided by the capital stock) with both measures of the variable. The translog simulation certainly captures the rise in profits that began after 1800, although it shows a curious blip in the eighteenth century.

Figure 11 repeats the exercise for the real rent of land. The history of rent has been the subject of considerable controversy, but the Norton, Trist, and Gilbert (1889), Allen (1992), and Clark (2002) series agree reasonably well for this period, as does the Turner, Beckett, and Afton (1997) series after 1800. Figure 11 shows Clark’s (2002, p. 303) series inclusive of taxes and rates. The simulated rent series closely follows this series.

The simulations show that the model reproduces the history of the important endogenous variables. This is important evidence for its validity and justifies the counterfactual simulations that will be discussed.

The production function and economic growth

The estimated production function has a property that plays an important role in explaining both economic growth and the two phase history of inequality. That property is the elasticity of substitution between capital and labour, which is close to zero. Figure 12
The translog function is not necessarily concave for all parameter values and input levels. The discerning reader may be able to see that the translog isoquant in Figure 12 turns up when capital increases from 600 to 800—in violation of the standard assumptions. This defect is an issue for only the last few years of one simulation discussed in this paper.

The actual 1810 capital-labour ratio and the ratio of the marginal products of labour and capital were first obtained. Labour was next increased by a small amount, which increased output in turn. Then capital was reduced until the original output was produced. The new capital-labour ratio and corresponding ratio of marginal products were computed. The elasticity of substitution was computed as the ratio of the percentage change in the capital-labour ratio divided by the percentage change in the marginal product ratio. The procedure was repeated with different changes in labour with little variation in the elasticity.

This result highlights the artificiality of standard growth accounting like Table 1. Growth accounting purports to identify ‘sources of growth’ that are additive and independent. We can vary one number in the table—the productivity growth rate or the capital stock—without varying others and add up the ‘effects’ to see how output would vary. That procedure is defensible only when the production function is Cobb-Douglas. When it is not, as in the present case, such counterfactual calculations can be very misleading.

The low elasticity of substitution also has implications for the debate about the growth rate during the industrial revolution. Williamson (1984) provocatively raised the question...
“Why was British Growth so Slow During the Industrial Revolution?” His answer emphasized the investment rate. “Britain tried to do two things at once—industrialize and fight expensive wars, and she simply did not have the resources to do both.” (Williamson 1984, p. 689). During the Napoleonic Wars, government borrowing crowded out private investment cutting the rate of accumulation and income growth. In contrast, Crafts (1987, p. 247) emphasized the slow pace of technical progress: “as a pioneer industrializer Britain found it hard to achieve rapid rates of productivity growth on a wide front throughout the economy.” Only if technology had advanced more rapidly could Britain have grown more rapidly.

The crowding out thesis has had a mixed reception. Heim and Mirowski (1987, 1991) have argued that British capital markets were too segmented for government borrowing to have crowded out private investment, a view that received some support from Buchinsky and Pollak (1993). Mokyr (1987) and Neal (1991, 1993) have argued that capital inflows offset government borrowing and precluded crowding out. Clark (nd) has found no impact of war finance on a variety of rates of return. On the other hand, Temin and Voth (2005) have inferred from the records of Hoare’s bank that crowding out probably occurred, although to a smaller extent than Williamson thought. Early in this debate Crafts (1987, p. 248) established the important point that private investment declined very little during the French Wars—the government borrowing represented additional savings provided by an aristocracy eager to defeat revolution and protect its position.

Whatever one concludes about crowding out, the possibility that a low British savings rate retarded the industrial revolution remains a live issue: What government borrowing during the French Wars indicates is that the propertied classes had a great, untapped potential to save. Indeed, a savings rate of 17% out of property income is remarkably low by international standards. From a regression like equation 13, David (1978) deduced that 61% of American property income was saved in the nineteenth century. We can, therefore, ask what impact a higher savings propensity would have had on growth. The question can be considered over a longer time frame than simply the French Wars, although it includes them. This is doubly fortunate since the stagnation in working class living standards was a much longer run affair than Williamson thought when he analysed the impact of the wars on accumulation.

To investigate the impact of savings on growth, I have simulated the model varying the savings rate out of property income starting in 1801. The effect depends on the magnitude and direction of the change. Doubling the savings rate, for instance, would have had only a modest impact on GDP per head. The simulated value in 1860 rises to £708 million from its baseline value of £661 million. Since increasing the savings rate (in this model) has only a small impact on growth, one cannot say that British growth was significantly reduced by a low savings rate. The underlying reason is that capital ran into rapidly diminishing returns given the low elasticity of substitution of the production function. Conversely, however, a cut in the savings rate below the historical level significantly reduces GDP. For instance, if the savings rate out of property income is cut to 40% of its actual value, then GDP growth is also cut to about 40% of the actual increment. Increases in the savings rate out of property income up to its historical level cause significant increases in the GDP gain. However, increases in the savings rate above the historical trajectory cause only minimal rises in GDP growth. While British capitalists and landowners had the capacity to save much more income, the implication of the simulations is that increments in GDP from more accumulation would not have justified the higher level of investment. The case that war related borrowing cut the
growth rate by crowding out private investment does not receive support from these simulations.

The history of inequality

While the integrated model of growth and distribution has important implications for recent debates on the sources of growth during the industrial revolution, its implications for the history of inequality are even greater. The model explains the two stage history of British inequality between 1760 and 1913 and offers a framework that identifies their causes.

Earlier I showed that the model simulated the history of output and wages up to 1860. Figure 13 shows that this success continued to the First World War. The upswing in the simulated real wage after mid-century is particularly noteworthy, for that change marks the shift from the first to the second stage of Lewis’ growth scenario.

While *grosso modo* the model reproduces the key trends in the British economy, it does not work quite as well in detail after 1860 as it did before. This is seen in Figure 14, which shows actual and simulated factor shares. The model simulates too high a value for the share of land and too low a value of the share of profits. This is not surprising since the model was calibrated over the period 1770-1860 when the British economy was comparatively closed. It became much more open after the middle of the nineteenth century with the repeal of the Corn Laws and Navigation Acts and the construction of a global system of railways and steamships. The ‘grain invasion’ of the late nineteenth century depressed British agriculture and rents (O’Rourke 1977). In addition, the opportunities for foreign investment increased dramatically, and millions of Brits moved to North America and Australasia. Growing openness meant that international factors played a much more dramatic role in growth and income distribution in Britain (O’Rourke and Williamson 2005). Since none of this is included in the model, it is not surprising that it does not track distributional shares as well as it did pre-1860. What is even more surprising, however, is that it works as well as it does. The implication is that technical progress and capital accumulation continued to play important roles in determining output and wages in Britain: cheap American food, in other words, was not the decisive reason that the real wage rose after 1870.

Why did Britain exhibit the two stage inequality history that Lewis highlighted? It was not for the reason he advanced, namely, the disappearance of surplus labour. Rather, balance was restored between the accumulation of capital and the growth of productivity.

– The first stage of rising inequality was precipitated by the acceleration of technical progress after 1800 in conjunction with the low elasticity of substitution between capital and labour in the aggregate production function. With technical progress specified as labour augmenting, a higher rate of technical progress was like more rapid population growth: it reduced the ratio of capital to augmented labour. A lower capital-labour ratio implied a higher marginal product of capital. With an elasticity of substitution less than one, the higher marginal product of capital translated into a higher share of capital in national income. Inequality increased and the real wage stagnated.

– The first stage contained the seeds of its own undoing, however. As the share of profits increased, the economy-wide savings rate rose since capitalists saved a constant share of their income. As a result, capital accumulation accelerated. Eventually, enough capital was accumulated to correspond to the requirements of higher productivity. Once steady state growth was achieved, so capital grew as rapidly as augmented labour, productivity growth
boosted the real wage as well as GDP per worker. This change occurred in the middle of the nineteenth century. Britain shifted from Lewis’ first stage to his second.

The transition from the first stage to the second, which occurred between the publication of the *Communist Manifesto* (1848) and volume I of *Capital* (1867), provides a wry commentary on Marx’s expectations. The acceleration of productivity growth did, indeed, shift income from workers to capitalists, as he expected. The result, however, was not continually increasing immiseration, for the capitalists invested a portion of their extra income and the increase in the capital stock eventually allowed rising productivity to be manifest as rising real wages. History did, indeed, exhibit a stage pattern of evolution, but the stage of flat real wages was followed by the most sustained rise in real wages ever seen—not by socialist revolution. The integrated growth model captures the logic of history.

**Malthus versus Marx**

The classical economists shared a common expectation that capital accumulation and technical progress would not trickle down to the working class as rising wages, but they disagreed about the reason for wage stagnation. Marx thought that a high rate of labour augmenting technical progress would reduce labour demand and keep wages from rising. Malthus, on the other hand, accepted that technical progress would increase the demand for labour but believed this would be offset by an increase in the population. We can explore these conjectures by simulating the model with different rates of productivity growth and population growth.

To explore Marx’s view, we can simulate the economy holding the rate of productivity growth at a pre-industrial level. In that case, there was no economic growth and no change in inequality. Rising productivity was a necessary condition for rising inequality—indeed, for anything at all to happen.

The result is more interesting if we simulate the industrial revolution and eliminate the population explosion that accompanied it. Figure 15 shows the trajectories of output per worker and the real wage from 1760 to 1860. Both tend upward. Engel’s pause in real wage growth is eliminated. Without the burden of equipping an expanding population, the increased demand for capital induced by rising productivity could be met without a marked shift of income to property owners. Consequently, population growth was a necessary condition for stationary real wages: Engels’ pause looks like Malthus’ dismal science come true.

History was more complicated, however. While population growth was a necessary condition for rising inequality, it was not sufficient. This is shown by the experience of Britain after 1860 when real wages rose in line with population even though population was growing as rapidly as in the first half of the nineteenth century. Population growth and technical progress were both necessary for an increase in inequality, but their impact was mediate by the adjustments to the capital stock that are at the core of the integrated growth model. Only by considering the feedbacks in the model can the evolution of output and wages be understood. Malthus and Marx are not enough.

**Conclusion**

The analysis of this paper changes the emphasis in our understanding of the industrial revolution. Three general revisions stand out. First, inequality rose substantially in the first four decades of the nineteenth century. The share of capital income expanded at the expense of both land and labour income. The average real wage stagnated, while the rate of profit
doubled. Second, the explanation of growth cannot be separated from the discussion of inequality since each influenced the other. In the first instance, it was the acceleration of productivity growth that led to the rise in inequality. Reciprocally, it was the rising share of profits that induced the savings that met the demand for capital and allowed output to expand. Third, the sources of growth cannot be partitioned into separate, additive ‘contributions’ in the manner of growth accounting. This procedure has always been counterintuitive to economic historians, for how could the productivity gains of machine spinning or iron puddling have been realized without capital investment? The complementarity of investment and greater efficiency is very clear in the model of this paper. Moreover, these two general points are interconnected: the production function parameters that make capital accumulation and technical progress complements in the growth analysis are implied by the change in the factor shares between 1770 and 1860.

With these general considerations in mind, we can outline the story of the industrial revolution as follows: The prime mover was technical progress beginning with the famous inventions of the eighteenth century including mechanical spinning, coke smelting, iron puddling, and the steam engine. It was only after 1800 that the revolutionized industries were large enough to affect the national economy. Their impact was reinforced by a supporting boost from rising agricultural productivity and further inventions like the power loom, the railroad, and the application of steam power more generally (Crafts 2004a). The application of these inventions led to a rise in demand for capital--for cities, housing, and infrastructure as well as for plant and equipment. Consequently, the rate of return rose and pushed up the share of profits in national income. With more income, capitalists saved more, but the response was limited, the capital-labour ratio rose only modestly, the urban environment suffered as cities were built on the cheap, and the purchasing power of wages stagnated (Williamson 1990). Real wages rising in line with the growth of labour productivity was not a viable option since income had to shift in favour of property owners in order for their savings to rise enough to allow the economy to take advantage of the new productivity raising methods. Hence, the upward leap in inequality.

The rise in inequality had ramifications that meant it was self-extinguishing. The increase in profits induced enough capital formation by the middle of the nineteenth century for the economy to realize a balanced growth path with capital and augmented labour growing at the same rate. Under this condition, the real wage grew in line with productivity. The European grain invasion and the chance to move to Australia, Canada, or the USA also boosted the real wage. They were not of fundamental importance, however: The burden of the integrated growth model is that productivity growth and capital accumulation were principally responsible for the rise in working class living standards after 1850, just as they had been responsible for their stagnation in the first half of the nineteenth century. Even sustained, rapid population growth was not enough to prevent labour incomes from rising once the accumulation conditions were right.
Table 1
Growth Accounting

<table>
<thead>
<tr>
<th>Growth of</th>
<th>Due to growth in:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y/L</td>
<td>K/L</td>
</tr>
<tr>
<td>1760-1800</td>
<td>.26%</td>
</tr>
<tr>
<td>1800-1830</td>
<td>.63</td>
</tr>
<tr>
<td>1830-1860</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Note:
The table shows growth rates per year for Y/L and A. The entries for K/L and T/L are the contributions of their growth to the growth in Y/L, that is the growth rates per year of K/L and T/L multiplied by the factor shares of capital (.35) and land (.15), respectively.
Table 2

Translog coefficients

\[\begin{align*}
\alpha_0 &= 0.481081 \\
\alpha_k &= 0.255594 \\
\alpha_l &= 0.518544 \\
\alpha_t &= 0.225862 \\
\beta_{kk} &= -1.58305 \\
\beta_{kl} &= 1.291068 \\
\beta_{kt} &= 0.291984 \\
\beta_{ll} &= -0.99908 \\
\beta_{lt} &= -0.29198 \\
\beta_{tt} &= 2.797242 \times 10^{-16}
\end{align*}\]

Note: These coefficients were computed after rescaling the labour and land indices to equal 248 in 1760, the same value as the capital stock in that year.
Figure 1

The two phases of the British Industrial Revolution

Engels’ Pause

<= Wage falling behind output growth  Wage rising with output =>

1760 1780 1800 1820 1840 1860 1880 1900 1920

historical GDP/worker  historical real wage
Figure 2

Historical Factor Shares, 1760-1860
Figure 3

Historical Profit Rate, 1760–1860

hist profit rate  Deane/Cole
Figure 4
Savings Propensity out of Property Income
Figure 5

Actual and Simulated GDP
Figure 6
Actual and Simulated Investment Rate
Figure 7
Labour’s Share of GDP: Actual and Simulated
Figure 8
Factor Shares: Historical and Simulated
Figure 9
Real Wage: Actual and Simulated
Figure 10
Profit Rate: Actual and Simulated
Figure 11
Land Rent: Actual and Simulated

1850 shillings per acre

1761 1781 1801 1821 1841

simulated actual
Figure 12
Cobb-Douglas and Translog Isoquants in 1810
Figure 13
Simulating Britain’s Two Stage Development Trajectory

- simulated GDP/worker
- simulated real wage
- historical GDP/worker
- historical real wage
Figure 14
Simulated GDP/head and the Real wage with no Population Growth
Figure 15

Simulated Factor Shares with no Population Growth
Appendix I: Data Description

We know much more about economic growth during the industrial revolution than was known fifty years ago thanks to the efforts of several generations of economic historians. Key variables, however, have only been established for benchmark years—real national income, in particular, has been estimated only for 1760, 1780, 1801, 1831, and 1860. The small number of observations precludes the econometric estimation of important relationships and requires calibrating the model instead. Also different series use different benchmark years. To bring them into conformity and to simplify simulations, all series are annualized by interpolating missing values. As a result, the series are artificially smoothed but capture the main trends. Real values are measured in the prices of 1850-60 or particular years in the decade as available. The price level did not change greatly in this period. All values apply to Great Britain unless otherwise noted.

Crafts and Harley have been continuously improving the measurement of British GDP (Crafts 1985, Crafts and Harley 1992, Harley 1993), and I have relied on their work. Based on Deane’s work, Feinstein (1978, p. 84) reckoned GDP in 1830 at £310 million and in 1860 at £650 million (both in 1851-60 prices), and I have extrapolated the 1830 estimate backwards using the Craft-Harley (1992, p. 715) real output index. This gives real GDP estimates for the benchmark years just noted.

The inputs were measured as follows:

- **Land**—acreage of arable, meadow, and improved pasture (commons are excluded). Allen (1994, p. 104, 2005) presents benchmark estimates for England and Wales. Following McCulloch (1847, Vol. I, pp. 554-5, 566-7), these have been increased by 12% to include Scotland.
- **Labour**—for 1801, 1811, and continuing at ten year intervals, Deane and Cole’s (1969, p. 143) estimates of the occupied population were used. The occupied population for 1760 was estimated by applying the 1801 ratio to the population. Voth (1991) has argued that the working year lengthened in this period. I have not tried to adjust the data for this change, so some of the rise in productivity that I report may be due to greater work intensity.
- **Capital** (and real gross investment)—Feinstein (1988b, p. 441) presents average annual gross investment by decade from 1760 to 1860 for Great Britain. The magnitudes are expressed in the prices of the 1850s. He also estimated the capital stock in the same prices at decade intervals by equation 3. To annualize the data, I assumed that real gross investment in each year equalled the average for its decade. I reconstructed the capital stock year by year with equation 3. With the annualized data, a depreciation rate of $\delta = 2.4\%$ per year gives a capital stock series that matches Feinstein’s almost exactly at decennial intervals. Therefore, 2.4% was used in subsequent simulations.
Appendix II: Results using Clark’s consumer price index

The model in this paper has been recalibrated using Clark’s consumer price index to measure the real wage. The elasticity of substitution in the production function changes little. The factor shares move less dramatically against labour than previously because the relative factor prices change less than before. Otherwise, the main results are very similar. I show graphs corresponding to Figures 1, 2, 3, and 13 in the main text.

Figure 1-Clark

The two phases of the British Industrial Revolution
Figure 2-Clark

Historical Factor Shares, 1760-1860
Figure 3-Clark

Historical Profit Rate, 1760-1860
Figure 13-Clark

Simulating Britain’s Two Stage Development Trajectory
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